Volume XI. *July, 1900.* *Number 1.*

THE

PHYSICAL REVIEW.

THERMODYNAMICS OF THE VOLTAIC CELL.

By Henry S. Carhart.

I. Theoretical.

THE theory, commonly known as Thomson's, that the electrical energy of a reversible Voltaic cell is the equivalent of the chemical energy transformed has been abandoned as an exact expression of the facts since the appearance of Helmholtz's paper on "Die Thermodynamik Chemischer Vorgänge,"[1] followed by the experimental corroboration of Jahn[2] and others. It is now well understood that the electrical energy output of a reversible cell may be either greater or less than the heat equivalent of the chemical reactions. It is greater when the temperature coefficient of the cell is positive, and it is less when it is negative. If such a cell be connected in a circuit with an external resistance so large in comparison with the internal resistance of the cell that the Joule's heat due to the internal resistance is vanishingly small, then heat must be added to the cell to keep its temperature constant when a current passes through it if its temperature coefficient is positive. On the contrary, if its temperature coefficient is negative, it must give up heat to surrounding bodies to remain in thermal equilibrium. In the former case all the potential chemical energy transformed goes over into electrical energy, and in addition some heat undergoes a similar transformation. In the latter case only a part

[1] Sitzungsber. der Akad. der Wiss. zu Berlin, I., 1882, p. 24.
[2] Wied. Ann., Vol. XXVIII., pp. 21 and 491, 1886.

of the transformed chemical energy becomes electrical in form; the remainder appears as heat. The calomel cell devised by Helmholtz is an example of a cell with a positive temperature coefficient and a converter of heat into electrical energy. The Clark standard cell, on the other hand, has a negative coefficient and gives up heat or grows warmer when functioning as a primary cell. In the Daniell cell the electrical energy is almost the exact equivalent of the difference between the heat of combination of the zinc sulphate formed and the copper sulphate decomposed.

The Helmholtz principle has hitherto been applied to a Voltaic cell in bulk, as it were. In the investigation which I am about to describe I have applied it to the cell in parts—that is, the positive and negative sides of a Voltaic cell are treated separately. In other words, I have considered the thermodynamics of a cell more in detail than has hitherto been done. This treatment may be regarded as an extension of the Helmholtz principle.

A reversible cell is one of which it is at least theoretically true that the electrical energy obtained from its action, when applied inversely so as to reverse the chemical operations, exactly suffices to restore the cell to its initial condition. Imagine the electromotive force of a Daniell cell compensated by a second inverse electromotive force. If the latter is reduced, a current passes through the Daniell in the normal direction, zinc goes into solution and copper is deposited. If the opposing electromotive force is made to exceed that of the Daniell, copper goes into solution and zinc is deposited. When equal quantities of electricity have passed in the two directions, the cell has been restored to its initial condition and the cycle is complete. Hence the laws of thermodynamics may be applied to such a reversible process.

Let T be the absolute temperature and dH the quantity of heat which must be applied to the cell or be taken from it in order to keep its temperature constant during the passage of a quantity of electricity dq. Also let u be the total energy contained in the cell. It is a function of the temperature of both the positive and negative sides of the cell, the cell being constructed so that the two sides may be separately considered. It is also a function of the quantity of electricity q passing through the cell.

Then from the principle of the conservation of energy (indicating the positive side of the cell by p and the negative by n subscript, and letting E be the electromotive force),

$$J \cdot dH = \left(\frac{\partial u}{\partial T}\right)_p dT + \left(\frac{\partial u}{\partial T}\right)_n dT + \left[\frac{\partial u}{\partial q} + E\right] dq. \qquad (1)$$

The change of entropy dS during the flow of dq is

$$dS = J\,\frac{dH}{T} = \frac{1}{T}\left[\left(\frac{\partial u}{\partial T}\right)_p dT + \left(\frac{\partial u}{\partial T}\right)_n dT\right] + \frac{1}{T}\left[\frac{\partial u}{\partial q} + E\right] dq. \qquad (2)$$

Hence

$$\frac{\partial S}{\partial T} = \frac{1}{T}\left[\left(\frac{\partial u}{\partial T}\right)_p + \left(\frac{\partial u}{\partial T}\right)_n\right] \text{ and } \frac{\partial S}{\partial q} = \frac{1}{T}\left[\frac{\partial u}{\partial q} + E\right]. \qquad (3)$$

Differentiating the first of equations (3) with respect to q and the second with respect to T,

$$\frac{\partial^2 S}{\partial T \cdot \partial q} = \frac{1}{T}\left[\left(\frac{\partial^2 u}{\partial T \cdot \partial q}\right)_p + \left(\frac{\partial^2 u}{\partial T \cdot \partial q}\right)_n\right]$$

$$= \frac{1}{T}\left[\left(\frac{\partial^2 u}{\partial T \cdot \partial q}\right)_p + \left(\frac{\partial^2 u}{\partial T \cdot \partial q}\right)_n + \left(\frac{\partial E}{\partial T}\right)_p + \left(\frac{\partial E}{\partial T}\right)_n\right] - \left[\frac{\partial u}{\partial q} + E\right]\frac{1}{T^2},$$

or

$$T\left[\left(\frac{\partial E}{\partial T}\right)_p + \left(\frac{\partial E}{\partial T}\right)_n\right] = \frac{\partial u}{\partial q} + E. \qquad (4)$$

Substituting in equation (1),

$$J \cdot dH = \left(\frac{\partial u}{\partial T}\right)_p dT + \left(\frac{\partial u}{\partial T}\right)_n dT + T\left[\left(\frac{\partial E}{\partial T}\right)_p + \left(\frac{\partial E}{\partial T}\right)_n\right] dq. \qquad (5)$$

This last equation is the heat, expressed mechanically, which must be added to the cell or abstracted from it to keep its temperature constant while the quantity of electricity dq is passing through it. In fact if dT is zero, the equation becomes

$$J \cdot dH = T\left[\left(\frac{\partial E}{\partial T}\right)_p + \left(\frac{\partial E}{\partial T}\right)_n\right] dq. \qquad (6)$$

$\left(\frac{\partial E}{\partial T}\right)_p$ is the temperature coefficient of the positive side of the cell, the temperature of the negative side remaining constant; $\left(\frac{\partial E}{\partial T}\right)_n$ is

the temperature coefficient of the negative side, the temperature of the positive side being maintained constant; the whole bracketed quantity is the temperature coefficient of the cell as a whole, neglecting any small coefficient due to the thermo-electromotive force at the junction of the two halves of the cell.

These positive and negative coefficients are the thermo-electromotive forces at the contacts between the two electrodes and the liquids bathing them respectively. Still neglecting the thermo-electromotive force at the junction between the two sides of the cell, the temperature coefficient of the entire cell is zero only when

$$\left(\frac{\partial E}{\partial T}\right)_p + \left(\frac{\partial E}{\partial T}\right)_n = 0,$$

that is, when the coefficients on the two sides are numerically equal and of opposite sign; or else when both are zero. The first condition is nearly realized in the Daniell cell. For $Cu \underset{\leftarrow}{-} CuSO_4$, $\left(\frac{\partial E}{\partial T}\right)_p$ is 0.00073; for $Zn \underset{\leftarrow}{-} ZnSO_4$, $\left(\frac{\partial E}{\partial T}\right)_n$ is 0.00079. The direction of these electromotive forces is in each case from the liquid to the metal across the heated contact. Hence the former gives to the cell a positive temperature coefficient, and the latter a negative one, while the coefficient of the cell in its entirety is negative and small, as I have shown elsewhere.[1]

A further consideration of equation (6) shows that the maintenance of a constant temperature on both sides of the cell during the flow of the quantity dq, requires that heat represented in mechanical measure by $\left(\frac{\partial E}{\partial T}\right)_p dq$ must be *added* to the positive side of the cell, and $\left(\frac{\partial E}{\partial T}\right)_n dq$ must be *withdrawn* from the negative side; or, in other words, these expressions represent the loss of heat on the positive side, and the gain on the negative respectively. Divide them by the thermal capacity of the two sides of the cell, and we have the *fall* of temperature on the positive side and the *rise* on the negative. These temperature changes are due to the thermal or Peltier elec-

[1] Primary Batteries, pp. 137–146.

tromotive forces at the two contacts. On the negative side of the Daniell cell, for example, the current from the zinc to the liquid flows against the Peltier electromotive force, and heat is generated. On the opposite side the current flows from the copper to the liquid, or with the Peltier electromotive force at the contact, and therefore heat is absorbed and converted into the energy of the current.

If a reverse current be sent through the cell, all the physical phenomena, except that of the Joule's heat, are reversed. The current on the positive side flows from the copper to the liquid, or against the thermo-electromotive force, and Peltier heat is generated. On the zinc side heat is absorbed. With a reverse current, therefore, the temperature of the positive side of the cell rises faster than that of the negative, while with a direct current the temperature of the negative side rises faster than that of the positive, that is, under practical conditions where the Joule's heat causes a rise of the temperature of the entire cell.

While equation (6) is perfectly general, the preceding conclusions are applicable only when the Peltier electromotive force at the contact of the electrode and the liquid is directed from the latter to the former across the heated junction, both on the positive and the negative side of the cell. There are some exceptions to this general rule. For example, the thermo-electromotive force of $Fe—FeSO_4$ is zero. Hence a cell consisting of $Fe—FeSO_4—CuSO_4—Cu$ has a positive temperature coefficient[1] due to the thermo-electromotive force on the copper side, and the Peltier phenomenon is confined to the positive side of the cell. In the case of the Voltaic series $Ni—NiSO_4—CuSO_4—Cu$, the two thermo-electromotive forces are in the same direction through the cell. The temperature coefficient is large—about 0.8 per cent., and heat is absorbed at both electrodes. Such a cell takes heat from the surroundings on both sides and converts it into electrical energy.

II. Experimental.

The general conclusions arrived at above I have not found it difficult to confirm qualitatively. It was only when quantitative de-

[1] A one-volt standard cell, Am. Jour Sci., Vol XLVI., p. 64.

terminations were attempted that serious difficulties were encountered. These I have in a large degree happily overcome. The results are highly interesting.

The Daniell cell was chosen as the most convenient one for the experimental work, particularly because the necessary constants pertaining to it are known with a good degree of accuracy.

The cell was made as follows: Two glass tubes 9.5 cm. long and 3 cm. in diameter were provided near the bottom with side tubes 18 mm. long and 18 mm. wide (Fig. 1). Heavy, tight-fitting rubber collars served to connect the two halves of the cell together, with a plate of fine unglazed porcelain between them as shown. The edges of the porcelain plate were covered with paraffin after the rubber collars had been bound together with fine iron wire. No difficulty was found in making a water-tight joint.

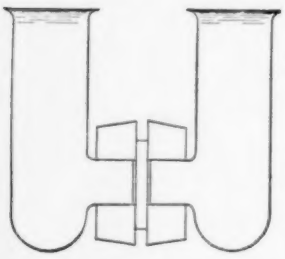

Fig. 1.

In most of the experiments 10 per cent. water-free solutions of zinc and copper sulphates were used. These solutions have relative conductances of 321 to 320.[1] They were chosen with the object of eliminating as far as possible the inequalities of Joule's heat between the two sides of the cell. In every experiment 40 c.c. of each solution were placed in the double cell, which was supported by a stout cord in a large pasteboard box and completely surrounded and covered with eider down.

The earlier observations were made with two Haak thermometers reading directly to fifths, and by means of a telescope to hundredths of a degree. The copper and zinc plates were attached directly to the thermometers by rubber bands, and the liquids were stirred before taking observations by moving the thermometers and the attached plates back and forth. Since the internal resistance of the cell was over 100 ohms, it was necessary to put it in series with a few storage cells and some resistance, with the double object of varying the current and of causing a reverse current to traverse the experimental cell.

[1] Kohlrausch and Holborn, Leitvermögen der Elektrolyte, p. 151.

The current was measured as follows: The electromotive force of a Weston standard cell was balanced against the fall of potential over a portion of two resistance boxes containing 10,000 ohms and connected in series with a storage cell. By means of an appropriate switch, consisting of mercury cups in a plate of paraffin, this potential drop in the auxiliary circuit was balanced against the fall of potential over the requisite number of ohms in the main circuit. For example, for 0.03 ampère the fall of potential over 34 ohms was thus balanced indirectly against the electromotive force of the Weston cell, which was very nearly 1.02 volts. The current was not allowed to vary more than about 0.05 per cent.

Fig. 2 shows the results obtained with a current of 0.02 ampère. Curve I, I, I, belongs to the zinc sulphate and II, II, II, to the cop-

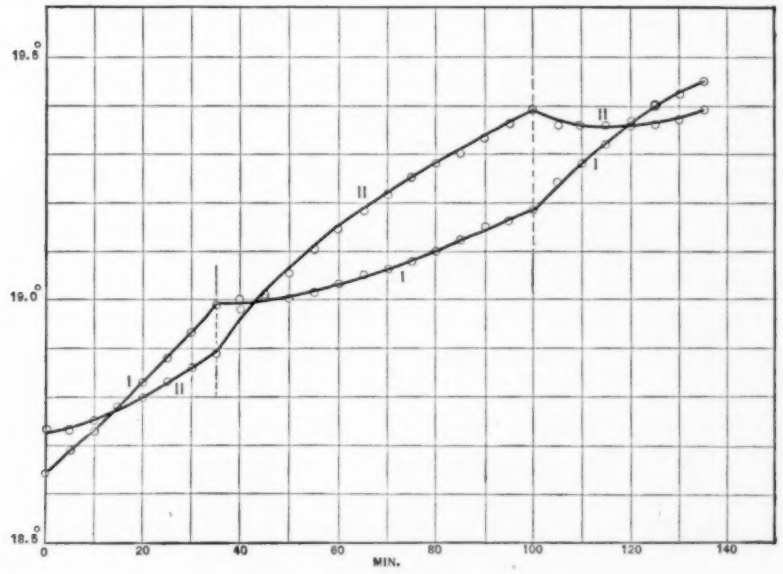

Fig. 2.

per sulphate, both of which in this case were fifteen-per-cent. solutions. These curves are valuable chiefly because they exhibit two facts rather strikingly. The current through the cell was reversed at the points indicated by the two vertical dotted lines. In the first place it is evident that the temperature always rises most rapidly on the side of the cell by which the current enters. The same fact

appears in every case and in every trial. In the second place the curves show how the phenomenon is complicated by the change in concentration. For this reason the difference in temperature between the two sides of the cell is less before reversal than it would otherwise be, for the concentration and conductance of the zinc sulphate increase while those of the copper sulphate decrease. After reversal this change in concentration accelerates the rise of temperature on the copper side and retards it on the zinc side. After reversing the current an established difference of temperature is reduced to zero in about half the time necessary to produce it. It is therefore obvious that some method had to be devised to eliminate the effect of this change of concentration as well as that of any difference in resistance on the two sides due to the dimensions of the apparatus, position of the plates, etc., if reliable quantitative results were to be obtained. For this purpose the following expedients were adopted: First, ten-per-cent. solutions of practically equal specific conductances were used. Second, the duration of any series of observations in either direction was taken inversely as the current, so that the same quantity of electricity was always transmitted with the same changes in concentration. This quantity was 72 coulombs,—0.03 ampère for 40 min., 0.04 ampère for 30 min., etc.

Third, a curve was constructed from observations showing the equalization of any established difference of temperature between the two sides of the cell as a function of the time. This may be called for convenience a "cooling curve." By means of this curve it is not difficult to correct any series of observations so as to obtain the temperature differences which would have been observed if heat had not flowed from one side to the other during the time covered by the series. Fourth, to eliminate the effect of the change in concentration and the lack of equal resistance on the two sides, a series of observations was first taken with a direct current through the cell, and then after reducing the temperature difference again to about zero without any current, another series of equal duration was taken with the current through the cell reversed. After correcting both series by the "cooling curve," the half sum of the two maximum differences of temperature was taken as the correct value. The

difference between the heats generated on the positive and negative sides, with the current first direct and then reversed, may be expressed by the two equations (R' being the resistance of the zinc side and R'' that of the copper),

$$J \cdot H' = I^2R't - I^2dR't + T\left(\frac{\partial E}{\partial T}\right)_n It$$

$$- \left[I^2R''t + I^2dR''t - T\left(\frac{\partial E}{\partial T}\right)_p It \right].$$

$$J \cdot H'' = I^2R't - I^2dR't - T\left(\frac{\partial E}{\partial T}\right)_n It$$

$$- \left[I^2R''t + I^2dR''t + T\left(\frac{\partial E}{\partial T}\right)_p It \right].$$

Half the algebraic differences of these two is

$$T\left(\frac{\partial E}{\partial T}\right)_p It + T\left(\frac{\partial E}{\partial T}\right)_n It = J \cdot dH,$$

and this is the quantity sought, considered arithmetically.

In the later experiments the thermometers were discarded and two thermal couples, consisting of thin iron and "constantan" wires, were substituted for them. The junctions were thickly covered with shellac and were then enclosed in thin glass tubes with enough mercury to cover the soldered junctions to aid in conductivity of heat. One junction was placed in the zinc sulphate side, the other in the copper sulphate. The plates of the cell were attached to the glass tubes containing the thermal junctions, and the tubes in turn were held by rubber bands to two long, thin rods of wood held firmly at their upper ends. The stirring of the liquids was thus easily accomplished by bending the wooden rods.

The galvanometer connected in series with the thermal couples was astatic and sufficiently sensitive, but its period was about 20 sec. The reading telescope was placed at a distance of 2.8 m., and the reduced deflection per degree difference of temperature was 138.3. The zero position was unfortunately a drifting one, so that the cross-wire of the telescope had to be set on it before every reading. The liquids were stirred for fifteen seconds before every reading. Five seconds intervened between the stirring and the observation.

With this arrangement I set out to determine, first, whether the temperature difference per second is proportional to the current, as equation (6) requires; second, how nearly the observed value of any established temperature difference agrees with the calculated value.

Fig. 3 shows the results of one series with a current of 0.05 ampère. Curve I belongs to the series of observations with a direct current, curve II to those with the current reversed, and III is the "cooling curve." Many other observations were taken for curve III, all of them agreeing well with this series.

Time in Min.	Reading, Deflection and Cooling Correction.					
	0.02		0.03		0.04	
0	3.6		4.0	0.2	1.0	
5	4.6		8.5	1.1	3.7	
10	6.0		11.0	1.4	5.0	0.2
15	6.4	0.2	12.5	1.4	7.5	0.6
20	7.3	0.4	14.0	1.6	8.6	0.9
25	7.7	0.5	15.0	1.7	10.0	1.1
30	8.0	0.6	16.2	1.8	11.1	
35	8.3	0.7	16.5	1.9		
40	8.5	0.8	16.8			
45	9.0	0.9				
50	9.1	1.0				
55	9.5	1.1				
60	10.0	6.4		12.8		10.1
Corrected Deflections,		12.6		23.9		12.9
REVERSE CURRENT.						
0	2.8		−0.2		3.3	0.2
5	6.9	0.7	3.0		8.7	1.2
10	10.0	1.2	5.0	0.2	13.2	1.6
15	12.0	1.3	7.7	0.3	16.9	2.1
20	13.3	1.5	9.2	1.1	20.0	2.4
25	14.4	1.6	10.9	1.4	22.8	2.8
30	15.5	1.8	12.0	1.5	24.8	
35	16.8	1.9	13.7	1.6		
40	17.2	2.0	14.6			
45	17.5	2.0				
50	17.8	2.1				
55	18.5	2.1				
60	18.6	15.8		14.8		21.5
Corrected Deflections,		34.0		21.4		31.8
Half Sum,		23.30		22.65		22.35

Time in Min.	Reading, Deflection and Cooling Correction.					
	0.05		0.05		0.06	
0	2.0		3.6		0	
4	5.0		5.8		4.1	
8	7.0	0.2	7.9	0.2	6.8	0.7
12	7.0	0.4	9.6	0.8	10.0	0.9
16	8.9	0.5	12.0	1.0	11.3	1.1
20	9.0	0.7	13.8	1.1	12.7	
24	10.8	1.0	15.4	1.2		
		8.8		11.8		12.7
Corrected Deflections,		11.6		16.1		15.4
REVERSE CURRENT.						
0	−4.0		4.8	0.3	1.0	
4	2.8		9.8	1.1	8.7	1.0
8	8.0	0.9	14.1	1.4	13.2	1.4
12	12.5	1.3	18.2	1.8	18.5	1.8
16	16.6	1.6	21.5	2.1	22.8	2.3
20	19.4	2.0	23.3	2.3	26.0	
24	23.0	27.0	26.0	21.2		25.0
Corrected Deflections,		32.8		30.2		31.5
Half Sum,		22.20		23.15		23.45
Mean of all,				22.85		

The above table contains all the essential data for the several currents used.

A deflection of 22.85 scale parts corresponds to a difference of temperature of 0°.165. If the phenomenon here measured is proportional to the current, then since the quantities transmitted with the above currents were all equal, the resulting corrected deflections should be equal. The results represented by the half sums show good agreement:

Current in Amperes.	Period in Min.	Half Sum.	Variation from Mean.
0.02	60	23.30	+0.45
0.03	40	22.65	−0.20
0.04	30	22.35	−0.50
0.05	24	22.20	−0.65
0.05	24	23.15	+0.30
0.06	20	23.45	+0.60

It is obvious that the heat generated on one side and the heat absorbed on the other are both proportional to the current in the case of a Daniell cell.

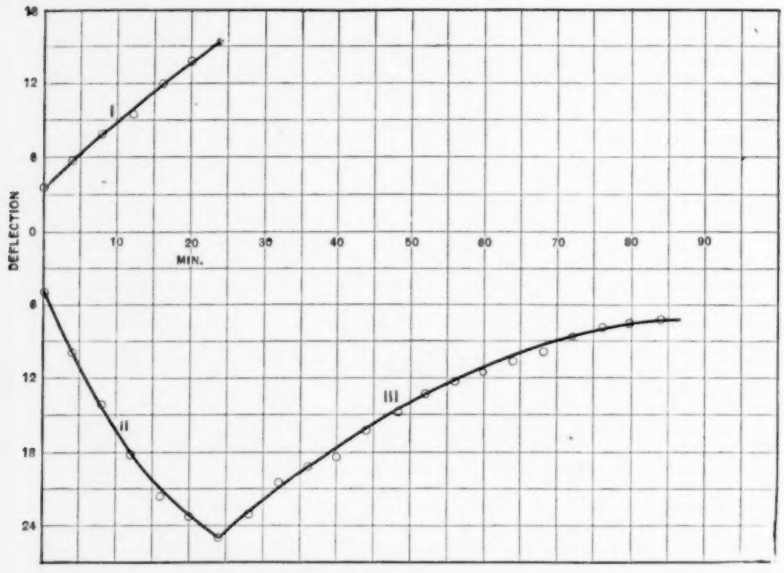

Fig. 3.

To calculate the difference of temperature between the two halves of the cell, it must be remembered that equation (6) expresses only the heat required to keep the temperature of the whole cell constant during the passage of a given quantity of electricity. To calculate the quantity corresponding to the observed mean of 22.85 or 0°.165, we must find the calories generated on one side of the cell and absorbed on the other and divide their sum by the thermal capacity of half the cell. For this purpose we have the following data:

$$1/J = 0.24 \qquad T = 292$$

$$\left(\frac{\partial E}{\partial T}\right)_n = 0.00079 \qquad \left(\frac{\partial E}{\partial T}\right)_p = 0.00073\,[1]$$

$$dq \text{ or } Q = 72.$$

Hence $H = 0.24 \times 0.00079 \times 292 \times 72 = 3.99$ (zinc side)

and $H = 0.24 \times 0.00073 \times 292 \times 72 = 3.68$ (copper side).

[1] Primary Batteries, pp. 140, 141.

The specific heat per unit volume of the solutions is 0.99. The mean of two determinations of the thermal capacity of the unfilled cell was 16.5. Hence the thermal capacity of one-half the cell is

$$40 \times 0.99 + 8.25 = 47.85.$$

The temperature difference produced by the passage of 72 coulombs should then be

$$(3.99 + 3.68) \div 47.85 = 0°.160.$$

The mean observed value is 0°.165.

I must reluctantly defer the application of the principles herein discussed to other types of cells for some later opportunity. It is obvious that the same method of investigation may be applied to a silver voltameter, a copper voltameter or a zinc voltameter. The essential fact involved in all is the presence of Peltier or thermo-electromotive forces.

My thanks are due to Professor H. F. Weber for his courtesy in welcoming me to his laboratory and in placing at my disposal the requisite apparatus for the work described in this paper.

ZÜRICH, April 3, 1900.

A NEW FORM OF ELECTRICAL CONDENSER HAVING A CAPACITY CAPABLE OF CONTINUOUS ADJUSTMENT.

LYMAN J. BRIGGS.

ADJUSTABLE condensers may be divided into two classes: The first class consists of condensers in which the capacity is varied by grouping a number of small capacities in series, or in multiple, or as a combination of the two methods, illustrated in the various forms of standard condensers having more than one stated capacity. Such condensers have the advantage of a known capacity for any grouping of the units which may be made. The adjustment is, however, limited to a comparatively few values and the method of adjustment, usually by plugs, is not well adapted for compensating capacity or inductance in a circuit, or for resonance work in general. Willyoung[1] has eliminated the disadvantages of adjustment by plugs in his adjustable condenser, in which a greater or less number of condensers of approximately equal capacity can be quickly thrown in multiple by a sliding switch. In all such arrangements the adjustment is, of course, not continuous. The first class may consequently be summarized as including such forms of condensers as are adjustable by steps.

The second class embraces those condensers which are capable of continuous adjustment, secured by changing the relative positions of the condenser plates. Sir William Thompson's sliding cylindrical condenser is an example of this class.[2] It consists of two conducting cylinders supported concentrically, one being capable of displacement with reference to the other along the common axis. A more familiar example is the classical flat disk air condenser, first used by Wilcke. These condensers leave nothing to be desired, so far as continuous adjustment is concerned. Their

[1] Condensers and their applications, p. 33. Published by Jas. G. Biddle, Phil., 1898.
[2] Absolute Measurements, Andrew Gray, Vol. I., p. 423.

large size, however, combined with a comparatively small maximum capacity, does not adapt them to laboratory requirements.

A stack of alternate conducting and insulating sheets (the conductors being so connected that adjacent sheets are of opposite polarity) gives the maximum capacity with reference to the space occupied. In condensers of this form the conducting and insulating materials used have been chosen heretofore so as to form a close, compact arrangement. If, however, we use an elastic conducting or insulating material which does not favor a compact arrangement of the plates, the capacity of the condenser may be varied at will by simply compressing the system. Such an arrangement combines the advantages of continuous adjustment with a condenser of considerable capacity arranged in the most compact manner, and is the principle employed in the construction of the condenser described in the present paper.

It is, of course, essential that the system of plates return to its original form when the pressure is removed. This is not readily secured through the elasticity of any of the ordinary insulating substances, but it can be accomplished by substituting for the tinfoil

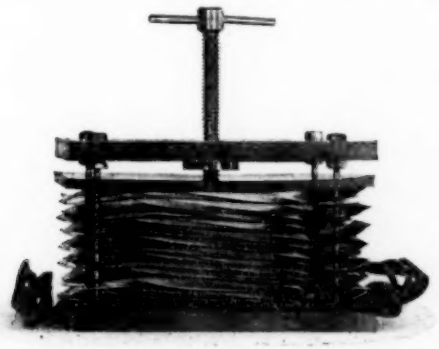

Fig. 1.

usually employed thin sheets of spring brass. To gain the necessary looseness of structure the sheets of brass are curved to conform to the surface of a cylinder of 6 or 8 inches radius. The curvature given to the sheet brass by the manufacturer in rolling it for shipment will usually be found to answer very well. Spring sheet brass, corresponding in thickness to No. 30 B. & S. wire

gauge, is suitable for condensers of small capacity, the thickness of the sheet depending somewhat upon the size of the condenser and the range desired. Mica is used for insulation.

A small condenser (.05 mf.) of the compression type is illustrated in Fig. 1. The outer end of each brass sheet projects one-fourth of an inch beyond the mica and is cut in the middle into two wings, one of which is bent downwards and the other upwards. The corresponding wings of alternate sheets are bent in opposite directions, so that when the sheets are assembled these wings come into contact, and electrical connection is established throughout the brass

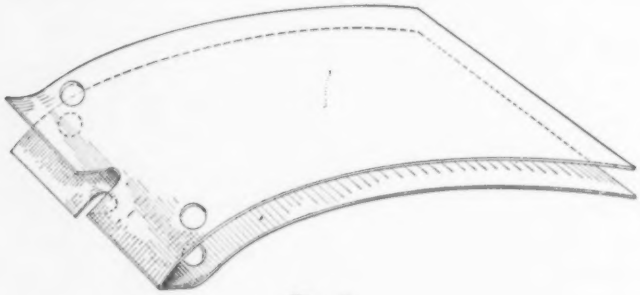

Fig. 2.

sheets of each polarity. The method of making contact between sheets of the same polarity is more clearly shown in Fig. 2. Owing to the movement of the wings when the system is compressed, this form of connection is virtually a rubbing contact, and gives good connection even when the condenser is immersed in oil. All necessity of soldering is thus avoided, and the condenser plates lie perfectly flat and smooth when the system is compressed.

Preliminary to assembling the condenser four holes are punched in the respective corners of each mica sheet, a template being used to secure similarity in all the plates. These holes are for the passage of the four guide or retaining rods on the condenser frame. Two similar holes are punched in the outer end of each brass sheet, the length of the brass sheet being so chosen that when the condenser is assembled the inner end of the sheet is at least one centimeter from the guide rods which carry the sheets of opposite polarity. The mica sheets are also two centimeters wider than the brass plates, to prevent leakage at the sides.

The compression frame consists of a base and top of hard rubber connected by the four retaining rods. A compression screw in the top, having a double thread of .1-inch pitch and operating against a compression plate of hard rubber resting on the top of condenser sheets, serves to adjust the capacity.

The assembling of the condenser is very simple, consisting merely in removing the top of the frame and stacking the sheets in their proper order upon the four guide rods. The structure is more open if the sheets representing the two polarities are assembled so as to have the convex surfaces face in opposite directions. Two mica sheets are used between each pair of plates.

The range in capacity which can be obtained depends upon the curvature, thickness and number of plates. In one condenser constructed the capacity when compressed was eight times the capacity when the pressure was released. The small condenser shown in Fig. 1, which is made up of thirteen plates on each side 5 by 8 cm. in effective dimensions, has a capacity in air of .055 of a microfarad when compressed, and about .013 microfarad when released. The maximum capacity of this condenser is thus over four times the minimum capacity. This ratio was found to be considerably greater in some of the other condensers constructed, depending upon the curvature and rigidity of the brass sheets.

The range of capacity may be greatly increased by dividing the condenser into three or four sections, so arranged as to permit their being thrown in multiple by a suitable switch, conveniently located on the top of the condenser frame. By dividing the condenser into three sections, the first of which contains three-fourths of the condenser sheets, the second three-fourths of the remainder or three-sixteenths of the whole, and the third section the remainder of the sheets, the range in capacity can be increased sixteen times, or the minimum capacity obtainable will be one sixty-fourth that of the maximum. The adjustment, moreover, will be perfectly continuous throughout this range. The capacity of the two smaller sections compressed will slightly exceed the capacity of all three sections released, while the capacity of the smallest section compressed will be greater than the capacity of the two smaller sections released. There will thus be no position between the limits through which a

continuous variation in capacity can not be obtained. The condenser terminals in this arrangement should of course be connected with the smallest section.

In constructing the condenser so as to permit the cutting out of a part of the plates to increase the range, it is necessary that the guide-rods should be insulated, as otherwise they may establish connection between the plates. This may be secured either by constructing them of hard rubber or by insulating metal guide-rods with thin-walled glass tubing of suitable size.

The range of the condenser in air can easily be increased temporarily by inserting a strip of mica between the wings of the condenser plates at any desired point. This method will often be found very convenient for increasing the range of the condenser when a switch for dividing the condenser into sections has not been provided.

The accompanying table gives the range in capacity which may be obtained with the small condenser shown in Fig. 1, by simply compressing the system of plates, none of the plates being cut out. The measurements were made by the direct deflection method, with an E. M. F. of 20 volts, comparing the deflections with those obtained from a standard condenser of approximately equal capacity.

TABLE I.

Change in Capacity with Expansion of the System of Condenser Plates.

Amount of Expansion Inches.	Capacity Microfarads.
0.00 (Compressed)	.0549
.05	.0497
.15	.0312
.25	.0258
.35	.0221
.45	.0205
.55	.0192
.75	.0175
.95	.0159
1.15	.0150
1.35	.0134

The data given in Table I. are expressed graphically in Fig. 3. The change in curvature at the upper end of the curve is due to the springing the condenser frame, giving erroneous values for the

amount of compression. It is to be noted that 90 per cent. of the variation in capacity occurs with but half of the total expansion shown, so that practically double the capacity could be used in a condenser frame of the same dimensions (10 by 7 by 5 cm.) without diminishing the range of adjustment to any extent.

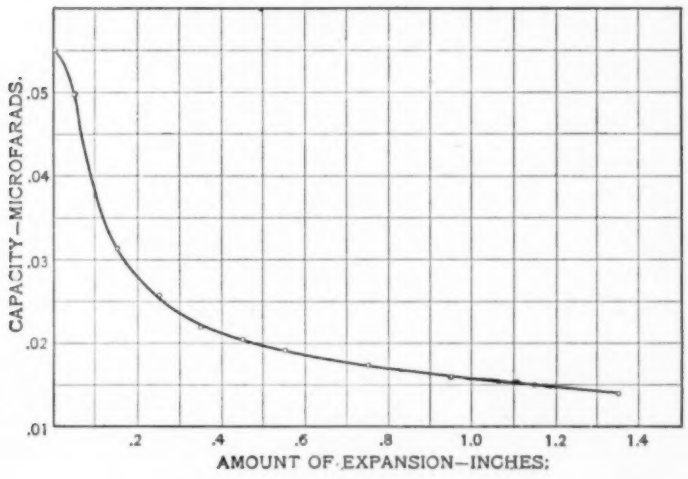

Fig. 3.

The results given in Table I. are the means of two series of determinations, one made while the stack of plates was being compressed and the other during its release. The two series of determinations corresponded very well, indicating an approximately constant capacity for any given position of the compression screw. The arbitrary setting of the condenser at a given capacity without comparison with a standard must necessarily be approximate, to some degree at least, owing to the liability of a slight re-arrangement of the condenser plates taking place, and is subsidiary to the real function of this form of condenser.

The capacity of the condenser is not simply a function of the distance between the plates, as would be the case if a homogeneous dielectric were used, but involves also the change in the specific inductive capacity of the combined dielectric—mica and air or mica and petroleum. The dielectric constants of air, petroleum and mica are approximately as 1, 2 and 6. Since the thickness of the

mica remains constant, the inductive capacity of the compound dielectric changes very rapidly in the first stages of the expansion of the stack of condenser plates. Consequently the change in capacity at this stage is much greater than that arising from the separation of the condenser plates alone. This is well illustrated in Fig. 3, which shows the change in capacity with expansion to be a function of the second degree, while with a homogeneous dielectric, the capacity is a linear function of the distance between the plates.[1] The greatest range of adjustment will consequently be attained by combining with mica the dielectric having the lowest specific inductive capacity.

When the condenser plates are entirely released the dielectric consists mainly of air or petroleum. The capacity of the condenser in this position would, therefore, be about twice as great in petroleum as in air and the range of adjustment in petroleum would consequently be only about one-half as much as in air. Experiments with the condenser in the two dielectrics confirmed these conclusions.

Fortunately the insulation resistance in air can be made sufficiently high so that an oil insulation is not necessary for ordinary voltages. The condenser when compressed in air successfully stood the continuous application of a direct E. M. F. of 225 volts, the highest E. M. F. available in the laboratory. The insulation resistance of the condenser when compressed in air was found to be about 5,500 megohms, determined by leakage. The insulation resistance is thus sufficiently high to permit the leakage being neglected in alternating current work.

An idea of the convenience and compactness of this condenser over other forms capable of continuous adjustment may be gained by comparing it with a parallel disk condenser of equal capacity. Assuming the mean distance between the disks to be 1 mm. the disks would have to be 275 cm. in diameter for a maximum capacity equal to that of the small condenser shown in Fig. 1 (.055 mf.).

This form of adjustable condenser was originally devised for the purpose of eliminating polarization capacity of the electrodes in an

[1]Absolute Measurements, Gray, Vol. I., p. 57.

electrical method of investigating the water content of soils *in situ*.[1] It has since proved convenient in compensating polarization capacity in electrolytic measurements in general. It is also adapted for use in other electrical measurements with alternating currents, such as correcting the displacement of the wave by induction, or in compensating capacity or induction in a bridge circuit.

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U. S. DEPARTMENT OF AGRICULTURE.

[1] Electrical instruments for investigating the moisture, temperature and soluble salt content of soils. Briggs, Bulletin No. 15, Division of Soils, U. S. Department of Agriculture, 1899.

This method of investigating the water content of soils depends upon the variation in the electrical resistance with changing moisture content. The soil resistance is measured between two carbon electrodes permanently buried in the soil at any desired depth. In order to secure a well-defined balance in the bridge it is necessary to counteract the capacity of the electrodes by shunting an equal capacity across that arm of the bridge which has a terminal in common with the soil electrode and the telephone receiver, so as to throw the two capacities on opposite sides of the bridge with respect to the receiver. Since the capacity of the bridge-arm embracing the soil resistance varied with different pairs of electrodes and with different conditions of the soil as regards moisture, it was necessary to have the compensating capacity adjustable and desirable to have the adjustment continuous throughout the required range.

ON THE THEORY OF THE COHERER.

BY KARL E. GUTHE AND AUGUSTUS TROWBRIDGE.

INTRODUCTION.

NO satisfactory explanation has as yet been advanced of the phenomenon of the lowering of the resistance of a loose contact when electromagnetic waves are set up in the surrounding medium, in spite of the numerous investigations made thereon. Of the existing theories, those of Branly, Lodge and Auerbach are the most important. Branly[1] supposes that the medium between the conducting particles of the coherer undergoes a modification under the influence of the electromagnetic waves and that thus the whole becomes conducting.

Lodge[2] tries to explain the phenomenon chiefly on electrostatic principles. He supposes the particles to be separated by films—for instance, oxide films. These films he calculates to be under enormous pressures, due to electrostatic attraction. Under these circumstances the surfaces may be welded together "especially if the electric stimulus simultaneously acted in any way as a flux, by reducing the infinitesimal tarnish of oxide or other compound which must be supposed normally to cover them."

The fact that Lodge compares the coherer phenomena to the welding of metallic spark gaps due to lightning discharges of leyden jars has led to an interpretation of his theory by later observers, that the spark plays the all-important rôle in producing an intimate contact between the particles, by the formation of metallic bridges.

Auerbach[3] advances what he calls a mechanical theory. He showed that by periodic vibrations of a mechanical nature a lowering of the resistance of a coherer takes place. He explains this as follows :

[1] C. R., 111, p. 785, 1890; 125, p. 939 and 1163, 1897.
[2] Phil. Mag., Vol. 37, p. 94, 1894; Electrician, Vol. 40, p. 87, 1897.
[3] Wied. Ann., Vol. 64, p. 611, 1898.

A disturbance is produced in the coherer of such a nature that each particle is displaced from its old position of equilibrium so that it comes into more intimate contact with its neighbor—it does not return to its former position, after the mechanical vibration, which caused its displacement, has ceased because of adhesion between the particles now in intimate contact.

He proposes to extend this explanation to coherer action in general.

Thus far very few experimental data are at hand to test any of the three theories outlined above. That of Branly can hardly be subjected to a crucial experimental test.

While Auerbach's theory seems probable enough for the rather strong mechanical vibrations he used, it can hardly be assumed that since the effect produced (viz: the lowering of the resistance) is the same in the mechanical and electrical case, the cause must be identical.[1]

A number of experimental proofs of Lodge's theory have been reported, notably those of Arons,[2] Tommasina[3] and Sundorph.[4]

The same criticism as the one advanced against Auerbach's theory holds here. It can very well be conceived that if sparks really do pass, a fusion of particles may take place, and the resistance be lowered—but the question still remains, "Is this the action of the coherer made use of in wireless telegraphy?"

If the lowering of the resistance be due to sparking, then we can only expect a very irregular behavior of the coherer. To quote Aschkinass who discusses Arons' and his own results in the following words:

"A lowering of the resistance takes place always when the excitation is weak, while no sparking, fusion or mechanical motion can be observed at the contacts. On the other hand, with stronger excitation, sparks, etc., can be seen, while the resistance is influenced in an irregular manner."[5]

He further states that when sparks of considerable intensity were

[1] See also Drude, Wied. Ann., 65, p. 486, 1898, and Aschkinass, ibid., 66, p. 306.
[2] Wied. Ann., 65, p. 567, 1898.
[3] C. R., 128, p. 1092, 1899; 127, p. 1014, 1898; 129, p. 40, 1899.
[4] Wied. Ann., 68, p. 594, 1899.
[5] Aschkinass, loc. cit., p. 290.

apparent, the resistance was sometimes lowered, sometimes increased, sometimes the changes were not permanent; often sparks could be seen without any change in resistance taking place at all.

From this it would appear that the phenomenon of coherer action can be best studied only when sparking is avoided. This point was thoroughly established by the quantitative experiments made by one of us a year ago[1] in which a remarkable regularity in the behavior of the coherer was observed, undoubtedly due to the fact that the above condition was fulfilled.

In making the experiments here reported, we were careful to avoid all disturbing influences, and here also the results are very concordant.

DESCRIPTION OF APPARATUS.

The lowering of the resistance of the coherer was produced by the closing of a circuit which consisted of a high E. M. F., which could be varied at will, a variable resistance R, and the coherer, in series.

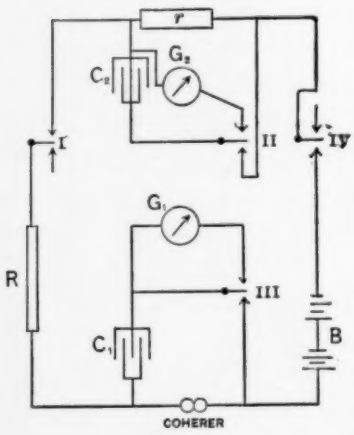

Fig. 1.

The resistance of the coherer was measured by observing simultaneously the current through, and the potential difference at the terminals of, the coherer.

In order to work always under the same conditions, the apparatus was so arranged that the above measurements weretaken always after the same very short time interval had elapsed since the circuit was closed. This could best be accomplished by the use of a pendulum which in short succession actuates four circuit keys.

I., II., III. and IV. are those keys. When the keys are set before the pendulum is released, they make contact on their under sides. The pendulum in swinging from left to right releases them so that they make contact on their upper sides.

[1] Trowbridge, Am. Jour. of Science, Sept., 1899.

As soon as the pendulum in swinging releases key I., the current flows through the coherer and the condensers C_1 and C_2 are charged. Then keys II. and III. are released simultaneously, which discharges the condensers C_1 and C_2 through the galvanometer G_1 and G_2 respectively; key IV. opens the circuit again. In general key I. was kept always in the same position relative to keys II. and III. However this was not necessary, for, as will be seen later, the time element does not enter into the problem. The galvanometers with their condensers, were calibrated beforehand so that the deflection of G_1 enabled us to determine the potential difference at the terminals of the coherer, that of G_2 the current flowing in the coherer circuit.

The coherers investigated by us were all ball coherers. The following form was found very convenient to work with.

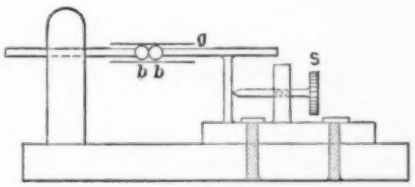

Fig. 2.

The balls *bb* were soldered to brass rods, one of which was in turn soldered to a stout metal post firmly embedded in the non-conducting base of the instrument. The other was supported by a brass spring, soldered to a plate which could be clamped to the base in any desired position. Against this spring a screw *S* could be pressed, allowing a fine adjustment of the contact.

In the investigation of a coherer of several contacts a glass tube *g* could be slipped over the balls to hold them in position.

In making observations, the method of procedure was the following: The coherer was adjusted by means of the screw so that it had a very high finite resistance. This resistance was roughly measured by means of an auxiliary circuit through the coherer, consisting of a battery of low electromotive force, a resistance and a galvanometer calibrated to indicate the current flowing in the auxiliary circuit.

This method was found to be preferable to one originally adopted by us, which was to set the contact by means of a microscope. By this method it was found difficult to get always an original high finite resistance, as sometimes when under the microscope the balls seemed to be in contact no coherer action took place on closing the

main circuit. By using the method of adjustment with the auxiliary circuit, coherer action always took place on working the pendulum apparatus.

In each series of experiments the applied voltage of the battery B was kept constant and with a given resistance $(R + r)$ in series with the coherer the pendulum was released, and the readings of galvanometers I. and II. were noted: the contact was then readjusted to its original high resistance, and the observations repeated from 10 to 15 times.

Next $(R + r)$ was varied and a similar set of observations taken.

The range of the values of $(R + r)$ was so chosen that the current through the coherer was varied from 0.002 ampère to 0.7 ampère.

For each coherer, several such series of observations were obtained with different applied voltages at B.

RESULTS.

A. *Experiments on coherers of a single contact.*

1. *Steel.*—Balls of three different diameters were used, 4.75 mm., 6.35 mm. and 9.5 mm.

In the following tables i denotes the current through the coherer observed with the galvanometer G_2, p the potential difference between the terminals of the coherer observed with the galvanometer G_1 and the resistance of the coherer calculated from p and i.

TABLE I.

Steel: Diameter = 4.75 *mm. Applied voltage* 6.5 *volts.*

i	p	ρ	i	p	ρ
0.0027	0.0457	16.59	0.0521	0.234	4.48
0.0054	0.0894	16.62	0.124	0.229	1.84
0.0106	0.174	16.40	0.254	0.237	0.94
0.0273	0.208	7.63	0.492	0.228	0.47

TABLE II.

Steel: Diameter = 4.75 *mm. Applied voltage* 14.0 *volts.*

i	p	ρ	i	p	ρ
0.0026	0.0218	8.42	0.124	0.224	1.84
0.0064	0.0451	7.10	0.212	0.229	1.08
0.0127	0.0904	7.12	0.309	0.228	0.74
0.0256	0.160	7.07	0.604	0.235	0.39
0.0621	0.211	3.40			

Table III.

Steel: Diameter = 4.75 *mm. Applied voltage* 60 *volts.*

i	ϕ	ρ	i	ϕ	ρ
0.0051	0.0103	2.01	0.0989	0.141	1.42
0.0124	0.0218	1.77	0.247	0.196	0.78
0.0247	0.0458	1.94	0.494	0.232	0.47
0.0494	0.0832	1.79	0.617	0.226	0.37

Table IV.

Steel: Diameter = 4.75 *mm. Applied voltage* 220 *volts.*

i	ϕ	ρ	i	ϕ	ρ
0.0056	0.0021	0.38	0.221	0.0873	0.39
0.0223	0.0114	0.51	0.313	0.144	0.45
0.0551	0.0255	0.46	0.438	0.224	0.51
0.0749	0.0374	0.50	0.722	0.218	0.30
0.111	0.0534	0.48			

Table V.

Steel: Diameter = 6.35 *mm. Applied voltage* = 51 *volts.*

i	ϕ	ρ	i	ϕ	ρ
0.0118	0.024	2.04	0.162	0.186	1.09
0.0236	0.058	2.46	0.219	0.224	1.02
0.0477	0.104	2.20	0.479	0.239	0.50
0.0954	0.160	1.68	0.784	0.240	0.37

Table VI.

Steel: Diameter = 6.35 *mm. Applied voltage* = 220 *volts.*

i	ϕ	ρ	i	ϕ	ρ
0.0208	0.0134	0.64	0.297	0.121	0.41
0.0512	0.0250	0.49	0.420	0.178	0.42
0.103	0.0450	0.44	0.523	0.190	0.36
0.210	0.0760	0.36	0.696	0.230	0.33

Table VII.

Steel: Diameter = 9.5 *mm. Applied voltage* = 51.5 *volts.*

i	ϕ	ρ	i	ϕ	ρ
0.0106	0.0260	2.46	0.171	0.213	1.25
0.0257	0.0686	2.66	0.255	0.225	0.88
0.0512	0.0935	1.83	0.510	0.216	0.42
0.0727	0.160	1.72	0.836	0.224	0.27
0.105	0.181	1.68			

2. *Copper.*—Copper plated steel balls, diameter 9.5 mm.

TABLE VIII.

Copper: Diameter = 9.5 *mm. Voltage applied* = 50 *volts.*

i	ρ'	ρ	i	ρ'	ρ
0.0101	0.004	0.41	0.512	0.171	0.33
0.051	0.041	0.80	0.819	0.174	0.21
0.102	0.081	0.78	1.252	0.171	0.15
0.256	0.154	0.60			

3. *Lead.*—Diameter 3.0 mm.

TABLE IX.

Lead: Diameter = 3.0 *mm. Applied voltage* = 14.2 *volts.*

i	ρ'	ρ	i	ρ'	ρ
0.012	0.020	1.67	0.118	0.120	1.06
0.024	0.040	1.67	0.291	0.121	0.42
0.059	0.081	1.53	0.573	0.123	0.22

TABLE X.

Lead: Diameter = 3.0 *mm. Applied voltage* = 39.5 *volts.*

i	ρ'	ρ	i	ρ'	ρ
0.009	0.009	1.00	0.120	0.107	0.90
0.018	0.020	1.11	0.182	0 131	0.72
0.036	0.036	1.00	0.361	0.129	0.36
0.073	0.069	0.94	0.591	0.131	0.22
0.092	0.087	0.94			

TABLE XI.

Lead: Diameter = 3.0 *mm. Applied voltage* = 220 *volts.*

i	ρ'	ρ	i	ρ'	ρ
0.021	0.008	0.39	0.433	0.085	0.19
0.051	0.016	0.31	0.512	0.101	0.19
0.102	0.036	0.35	0.683	0.129	0.17
0.209	0.060	0.29	1.024	0.127	0.12

4. *Phosphor Bronze.*—Diameter 3.2 mm.

TABLE XII.

Phosphor Bronze: Diameter = 3.2 *mm. Applied voltage* = 220 *volts.*

i	ρ'	ρ	i	ρ'	ρ
0.022	0.024	1.07	0.317	0.182	0.60
0.055	0.032	0.58	0.442	0.189	0.43
0.110	0.072	0.64	0.727	0.197	0.29
0.222	0.112	0.51			

From the foregoing tables it appears that, for increasing i, p approaches asymptotically a finite maximum value. This value is different for different substances but the same for any one substance whatever the applied electromotive force and, at least in the case of steel, it is practically independent of the size of the balls.

We have plotted the results contained in tables I. to IV. in Fig. 3, those of tables V. and VI. in Fig. 4 and those of tables IX. to XI. in Fig. 5.

In these figures the abscissas are the currents and the ordinates the corresponding values of the terminal potential differences at the coherer.

The simplest differential equation which fulfills the condition that p approaches a finite maximum value P is $dp = k(P - p)pi$ where k is a constant whose dimensions must be an inverse of a current.

This differential equation when integrated gives $p = P(1 - e^{-ki})$ Eq. 1, and, as will be shown later on, this exponential equation expresses analytically almost all of our results.

It seems perhaps to be more than a mere coincidence that this formula is identical with that which one of us (Guthe, PHYS. REV., Vol. 7, p. 193, 1898) has shown to hold for the polarization of copper in copper sulphate solution.

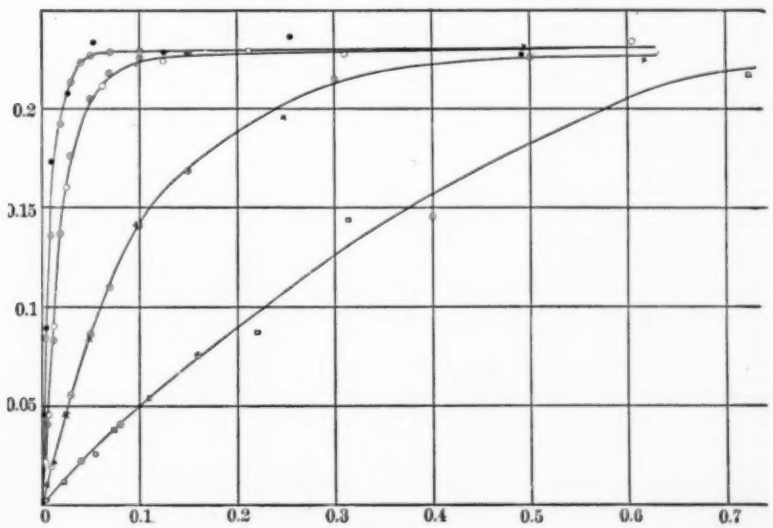

Fig. 3.

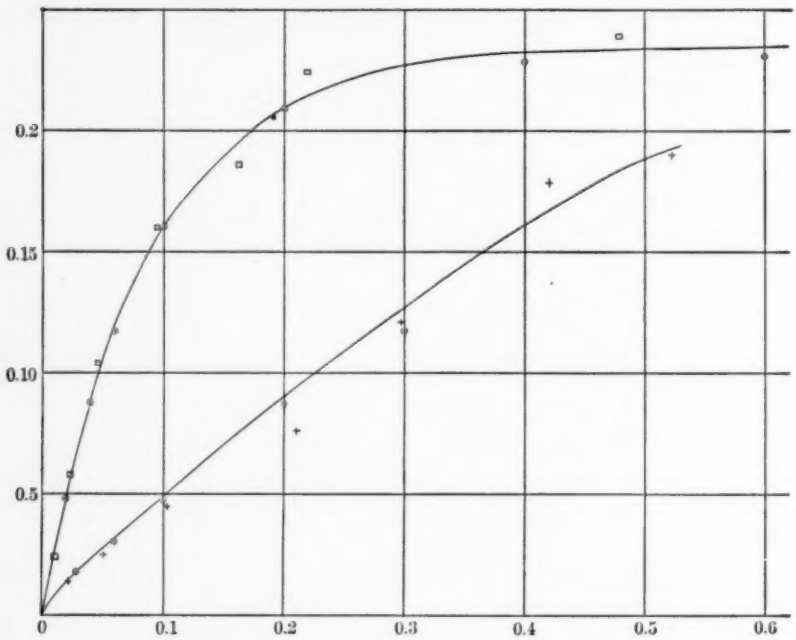

Fig. 4.

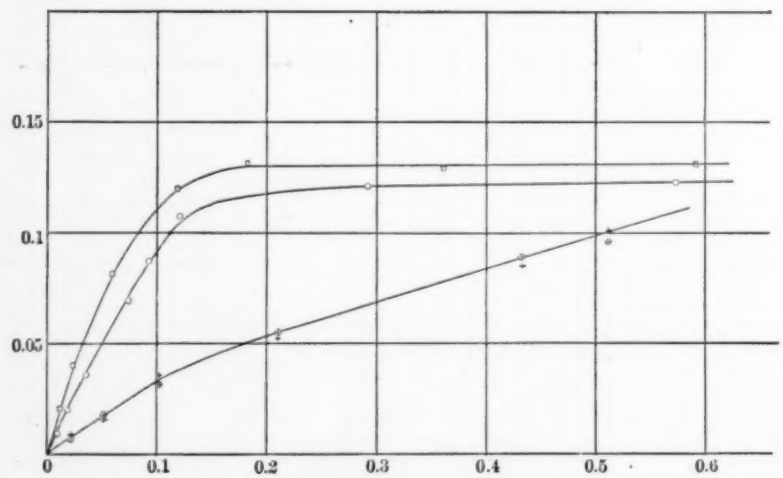

Fig. 5.

May not coherer action be explained in the following way? On the surface of the metals in loose contact we have a con-

densed, badly conducting atmosphere, possibly water vapor. Thus before a current flows the resistance is very high.

For simplicity let us consider two spherical conductors in loose contact with these films between them. As soon as potential difference exists between them the films are squeezed together by electrostatic attraction and over a circular area A the thickness of the films will be of molecular dimensions.

In Fig. 6, using the same coördinates as in the foregoing figures, the tangent of OT would represent the resistance of the film A at the moment the voltage is applied. As the current flows ions from the positive electrode break through the film forming metallic contact and thus reducing the resistance.

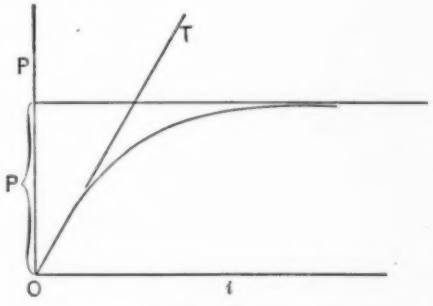

Fig. 6.

After the current stops flowing the balls are in intimate contact and held together by cohesion. Therefore the resistance does not increase again after the current ceases to flow.

That the resistance after lowering is not affected by a current *smaller* than that which produced the lowering was found to be the case by a series of experiments, while if a *larger* current were sent through, the resistance adjusted itself to the value it would have had had we started with a very high original resistance and sent through it this larger current.

In general we found that it makes very little difference what pressure the coherer is under before the current is sent through provided always the pressure is varied within reasonable limits.

As mentioned above the coherer particles under electrostatic pressure are in contact to within molecular distances over a certain area A. Now we make the supposition that a definite minimum number of ions is needed to carry any given current i.

Let P be the difference of potential over a film A filled with a number of ions just sufficient to carry the current i. Or in other P is the potential difference over a molecular layer of ions in which

there flows through each ion the maximum current it can carry. If the current be increased beyond this point one of two things might happen. Either the area A can be increased by the ions filling out a space exterior to A and the current be now carried by an area proportional to it or if the area cannot change then heating and perhaps melting might take place.

The area A defined above we will in future call the *critical area* and we will assume that the first of the above possibilities is what takes place in the coherer as it explains the horizontal part of our curves where $P =$ const.

Suppose now in the case of a given current i the area of the film of molecular thickness to be larger than the one which, when filled with ions, would carry this current. A greater than the *critical number* of ions will break through the surface in the first impulsive rush when the voltage is applied. In consequence the difference of potential produced will be less than the critical value P.

If n ions would just carry the current i we would have an ions in the case of a larger surface where a is greater than unity and will increase with the size of the contact surface.

For the critical surface the potential difference would be, according to definition, $P = \frac{r}{n} i$, for a surface larger than the critical $P = \frac{r}{an} i$, *i. e.*, $p < P$. r would be the ohmic resistance of an ion and a can be determined from our differential equation.[1]

This would mean that the nearer filled up the area is the less readily the ions will go over and hence the increase of p with increasing i will become smaller and smaller until it finally becomes zero when P is reached.

A rather striking relationship between the critical voltages P of the substances investigated by us suggests itself.

From the tables above we have P steel 0.23 volts, P copper 0.173 volts, P lead 0.127 volts. Supposing the valency of Fe and

[1] In case of surfaces of very large dimensions as in ordinary metallic contact then a will approach ∞ and the resistance of the contact which is $-r/an$ would approach zero. Now $dp/di = kri/n(1 - 1/a)$ and i/n is a constant. If a is so large that $1/a$ can be neglected we have the case of a continuous conductor and our differential equation reduces to Ohm's law.

Cu to be 2 and of Pb to be 4 then the gramme equivalents are respectively 28, 31.75 and 51.75.

Multiplying P Fe by 28 gives 6.44,
P Cu by 31.75 gives 5.50,
P Pb by 51.75 gives 6.57,

or $P \times$ gramme equivalent = const. nearly.

When we come to consider the curves for different applied electromotive forces we have to expect different electrostatic attraction between the balls to begin with.

Let us assume Hooke's law to hold for the coherer substance and that no shearing takes place. Let S be the area on which the pressure π acts, then $d\pi = a' S dl$ where dl is the displacement produced and a' is a constant. For displacements as small as we must assume them here to be $dS \sim dl$ from which follows $d\pi = a S dS$ or the total pressure $\pi = a/2 S^2$, Eq. 2.

Now to the same order of approximation π varies as the square of the potential gradient; therefore, very nearly, S varies directly as E. But the difference of potential necessary to bring the coherer balls into point contact is P (according to the assumption made above) hence if the applied voltage be called ES varies as $(E - P)$.

As shown in Fig. 6 the tangents of the curves at the origin are a measure of the resistance of the areas S. Now since for (lim $i = 0$) all our curves are straight lines the law of resistances must hold for the films at this point and ρ must be inversely proportional to S.

If this is true then the general form of our differential equation is $dp = c(P - p)1/(E - P)di$, Eq. 3.

The tangent for any curve, $E =$ const. for (lim $i = 0$) is $CP/(E - P) = kP$. As mentioned above this is the resistance of the film between the balls when the voltage E is applied. $k(E - P)$ should then be constant for one and the same substance. That this is so will be shown in Table XIX. In the tables preceding this one we have compared the numerical values calculated by means of Eq. 1 with the experimentally determined ones from our curves.

The results with phosphor bronze were rather irregular as compared with those for steel and copper.

TABLE XIII.

Steel: Small Diameter. Appli d voltage = 6.5 *volts.* $k = 90$. $P = 0.23\ v$.

i	$\hat{p}$ from curve.	$\hat{p}$ calculated.	i	$\hat{p}$ from curve.	$\hat{p}$ calculated.
0.005	0.078	0.083	0.04	0.223	0.223
0.01	0.136	0.136	0.05	0.227	0.226
0.02	0.192	0.191	0.07	0.229	0.228
0.03	0.213	0.212	0.10	0.229	0.229

TABLE XIV.

Steel: Small Diameter. Applied voltage = 14.0 *volts.* $k = 45$. $P = 0.23\ v$.

i	$\hat{p}$ from curve.	$\hat{p}$ calculated.	i	$\hat{p}$ from curve.	$\hat{p}$ calculated.
0.005	0.040	0.042	0.05	0.205	0.205
0.01	0.080	0.083	0.07	0.218	0.219
0.02	0.137	0.136	0.10	0.223	0.226
0.03	0.170	0.176	0.15	0.228	0.229

TABLE XV.

Steel: Small Diameter. Applied voltage 60 *volts.* $k = 9.5$. $P = 0.23\ v$.

i	$\hat{p}$ from curve.	$\hat{p}$ calculated.	i	$\hat{p}$ from curve.	$\hat{p}$ calculated.
0.01	0.020	0.021	0.10	0.141	0.140
0.03	0.056	0.057	0.15	0.169	0.168
0.05	0.086	0.087	0.3	0.214	0.216
0.07	0.110	0.111	0.5	0.227	0.227

TABLE XVI.

Steel: Small Diameter. Applied voltage 220 *volts.* $k = 2.5$. $P = 0.23\ v$.

i	$\hat{p}$ from curve.	$\hat{p}$ calculated.	i	$\hat{p}$ from curve.	$\hat{p}$ calculated.
0.04	0.022	0.022	0.4	0.145	0.145
0.08	0.041	0.041	0.8	0.198	0.198
0.16	0.075	0.076			

TABLE XVII.

Steel: Medium Diameter. Applied voltage = 51 *volts.* $k = 12$. $P = 0.23\ v$.

i	$\hat{p}$ from curve.	$\hat{p}$ calculated.	i	$\hat{p}$ from curve.	$\hat{p}$ calculated.
0.02	0.048	0.048	0.2	0.210	0.209
0.04	0.087	0.087	0.4	0.232	0.228
0.06	0.120	0.117	0.6	0.235	0.230
0.1	0.160	0.160			

TABLE XVIII.

Steel: Medium Diameter. Applied voltage = 220 volts. $k = 2.4$. $P = 0.23$ v.

i	$\hat{p}$ from curve.	$\hat{p}$ calculated.	i	$\hat{p}$ from curve.	$\hat{p}$ calculated.
0.03	0.018	0.018	0.2	0.090	0.087
0.06	0.030	0.030	0.3	0.125	0.117
0.1	0.048	0.048			

TABLE XIX.

Summary for Steel Balls.

(a) Small Diameter.			(b) Medium Diameter.		
E	k	$k(E-P)$	E	k	$k(E-P)$
6.5	90	564	51	12	610
14.0	45	619	220	2.4	528
60.0	9.5	568			Mean 569
220.0	2.5	550			
		Mean 575			

TABLE XX.

Copper: Applied voltage = 50 v. $k = 6$. $P = 0.171$ v.

i	$\hat{p}$ from curve.	$\hat{p}$ calculated.	i	$\hat{p}$ from curve.	$\hat{p}$ calculated.
0.03	0.028	0.028	0.2	0.123	0.120
0.06	0.050	0.051	0.3	0.150	0.156
0.1	0.081	0.080	0.6	0.172	0.171

$k(E-P) = 298.$

The curves for lead, while they have the same general form as the others, cannot be represented by our equation 1 except in the case where $E = 220$ volts. Certainly we cannot expect very concordant results with a substance as inelastic as lead. The lead balls on examination after use showed that they had undergone a permanent deformation.

TABLE XXI.

Lead: Applied voltage = 220 v. $k = 2.76$. $P = 0.127$ v.

i	$\hat{p}$ observed.	$\hat{p}$ calculated.	i	$\hat{p}$ observed.	$\hat{p}$ calculated.
0.021	0.0081	0.0071	0.433	0.0848	0.885
0.051	0.0161	0.0168	0.512	0.101	0.961
0.102	0.0363	0.0312	1.024	0.127	0.119
0.208	0.0525	0.553			

$k(E-P) = 607.$

In the curves given above the calculated values are plotted with the observed and marked by a ⊕ In general the agreement seems to be very satisfactory.

We satisfied ourselves that we have here to do not with a time-rate of heaping up of ions until a maximum is reached ($dp \sim dq$) but rather that the potential difference depends, in the main at least, on the current. This we did by changing the position of key I. (see Fig. 1) thus varying the time during which the current flowed within a considerable range without any appreciable effect on the value of p.

ADDITIONAL RESULTS.

Besides the above observations on coherers of a single contact we made a test of the effects of the number of contacts on the critical voltage P.

From the foregoing theory we must expect that the formula for a coherer of n contacts our differential equation must be

$$p = nP(1 - e^{-ki})$$

for if we have n contacts our differential equation must be

$$dp = k(nP - p)di$$

or we would expect that the potential difference is additive in character.

TABLE XXII.

Steel: Small Diameter. 5 *contacts.* *Applied voltage* = 15.2 *v.*

i	p	ρ	i	p	ρ
0.00363	0.228	62.8	0.130	1.258	9.7
0.00744	0.499	60.4	0.630	1.216	1.9
0.0142	0.832	58.6			

TABLE XXIII.

Steel: Small Diameter. 5 *contacts.* *Applied voltage* = 51.75 *v.*

i	p	ρ	i	p	ρ
0.0127	0.156	12.3	0.101	0.915	9.1
0.0254	0.339	13.5	0.251	1.210	4.8
0.0508	0.524	10.3	0.499	1.227	2.5

TABLE XXIV.

Steel: Small Diameter. 5 *contacts.* *Applied voltage* = 220 *v.*

i	p	ρ	i	p	ρ
0.00565	0.0166	2.94	0.223	0.478	2.14
0.0223	0.0686	3.08	0.316	0.707	2.23
0.0551	0.145	2.63	0.442	0.998	2.27
0.111	0.305	2.74	0.722	1.112	1.53

TABLE XXV.

Steel: Small Diameter. 10 *contacts.* *Applied voltage* = 220 *v.*

i	p	ρ	i	p	ρ
0.0226	0.143	6.33	0.313	1.684	5.38
0.0551	0.324	5.88	0.438	2.200	5.02
0.110	0.665	6.03	0.727	2.270	3.12
0.221	1.112	5.03			

The above tables are sample ones from the experiments on several contacts. We also investigated lead and phosphor bronze and found them to exhibit the same behavior, *i. e.*, $p_n = np_1$.

It may be mentioned here that in working with coherers it is advisable to have the coherer substance always in the same condition as regards oxidation. This has been pointed out by other observers. We usually cleaned the balls with the finest grade of emery paper and then let them stand for several hours before using. If used directly after cleaning a high initial resistance could not be obtained when the balls were in contact.

After several days' use a marked oxidation was observed to have set in and the critical voltage was found to have become somewhat higher.

The following example brings out this point. A set of balls freshly set up gave the following critical voltages:

One contact, 0.232 volt.

Two contacts, 0.468 volt.

Four contacts, 0.936 volt.

After a few weeks the values were:

Five contacts, 1.43 volts.

Ten contacts, 2.85 volts.

Twenty contacts, 5.84 volts.

The effect of oxidation may be that the badly conducting oxide particles make our hypothetical film between the balls thicker and hence the resistance of the film greater and the terminal potential differences also greater.

It may be of interest to point out the similarity with the behavior of electrolytic cells. Copper in copper sulphate after the copper is slightly oxidized shows a very much higher value of P than when the electrodes are clean.[1]

To satisfy ourselves that the coherers used were sensitive to Hertz radiations we set up a Hertz oscillator in the same room with the apparatus described above. The coherer was connected neither to earth nor to a vertical wire.

The resistance of the coherer was measured before and after the lowering due to the radiation by the method employed throughout this investigation, being careful, however, to use a current so small that it alone would produce no further lowering of the resistance of the coherer after radiation.

The Hertz oscillator was placed at distances varying from 2 m. to 7 m. from the coherer. The lower limits of the resistance were wholly independent of the original high resistance and decreased not quite as fast as the square of the distance.

Conclusion.

From the present investigation and from the work of other observers it seems to be justifiable to attribute the high original resistance of the coherer to the film on the surface of the metallic particles.

Dorn has shown that very little or no coherer action takes place between surfaces of the noble metals when they have been heated in a vacuum so as to expel as far as possible the surface film.[2]

He also found a decided decrease of resistance for other metals.

Aschkinass,[3] on the other hand, tried to remove the surface film from the filing particles in a tube by exhausting and heating but did not observe any change in the behavior of the coherer. He admits himself, however, that this can hardly be considered a decisive test.

[1] See Guthe and Atkins, Proc. A. A. A. S., 1899, p. 109.

[2] Dorn, Wied. Ann., 66.

[3] Loc. cit.

The fact mentioned above that metallic surfaces after thorough cleaning do not show a high contact resistance is also in favor of our supposition and we believe that Auerbach's results can easily be explained by the rubbing away of the films by mechanical action.

The films must form a sort of protecting layer on the metal surface as the following experiment shows:

Two coherers, consisting of one contact each, were put in parallel and the circuit closed through them. The resistance of each was measured before and after lowering and it was found that coherer action took place in one only. After standing a day this one was always oxidized while the other showed a bright surface where the balls had been in contact. The coherer action had apparently destroyed the protecting film of the one in which it took place.

Aschkinass mentions his interesting results with coherers consisting of PbO_2 or CuS particles as a proof against Lodge's theory. They behave in the opposite way from the ordinary coherer.

We would suggest in view of the similarity with polarization that the compounds are in this case broken up into two components, one of which conducts well and the other poorly. No actual metallic contact need then occur and the S or O would play a decided rôle in the increase of the resistance.

It seems that particles of any conducting substance may form a coherer—among others carbon. This suggests an interesting question in connection with the electric arc.

Ayrton and Perry[1] found that the difference of potential between the two carbons of an arc lamp was independent of the current strength provided the distance between them was kept constant, *i. e.*, the *apparent* resistance of the arc was inversely as the current.

Does this correspond to the horizontal line of our curves and does cohesion play any part in the starting of the arc?

We offer the foregoing considerations merely as a tentative explanation of this little-understood phenomenon, being well aware that some of our assumptions are somewhat crude, but hoping nevertheless that they may lead to a fuller understanding of the subject since they seem to furnish a satisfactory hypothesis.

PHYSICAL LABORATORY OF THE UNIVERSITY OF MICHIGAN, June, 1900.

[1] Phil. Mag., May, 1883.

ELECTRICAL RESISTANCE OF THIN FILMS DEPOSITED BY KATHODE DISCHARGE.

BY A. C. LONGDEN.

INTRODUCTION.

IF standard high resistances of great precision and unvarying values could be obtained at low cost, their application in the determination of insulation resistance by the direct deflection method would soon cease to be their sole field of general usefulness. The condenser and ballistic galvanometer would no longer be regarded as important or even desirable in comparing electromotive forces, and a high resistance could even be used with some advantage in place of a condenser, in the condenser method, for measuring internal battery resistance.

The use of numerous shunts in the determination of figure of merit is always regarded as something to be endured rather than desired. With a suitable high resistance in series with the galvanometer and standard cell, the determination of figure of merit becomes absolutely simple. The range of the Wheatstone bridge may also be enormously increased by the use of high resistances as bridge arms, and heavy currents may be measured with delicate galvanometers in series with high resistances, without making the resistance of the shunt through which the main current passes so small that the percentage of error in the calculations shall be large.

The use of carbon as a high resistance material is tolerably satisfactory for some purposes, if we are willing to re-standardize our resistances every time we use them, and to reckon with the enormously high temperature coefficient of the material; even then, the uncertainty of the contacts in most forms of carbon resistances is so great as to condemn such resistances in all cases where anything like careful or accurate work is contemplated.

A number of forms of carbon resistances have been used in connection with the research which furnishes the subject matter of this

article. It would not be in place to give detailed descriptions of them here, but it may be well to say in passing, that those which were the most nearly trustworthy consisted of sticks of pipe clay saturated with sugar solutions, of different degrees of concentration —the sugar being subsequently carbonized in the sticks by continued exposure to a red heat, in the absence of oxygen. The resistance of the stick depends upon the degree of concentration of the sugar solution used in preparing it.

While these resistances were very satisfactory for carbon, it must still be said that no carbon resistance can be considered for a moment in comparison with standard wire resistances.

In wire resistances, the use of alloys instead of pure metals is based upon the fact that alloys have, in general, lower temperature coefficients and higher specific resistance than pure metals ; but it must be borne in mind that, so far as high specific resistance is concerned, it is not in itself an advantage, but is only a means to an end. A wire, having a high specific resistance, enables us to obtain a high resistance, having small weight, small bulk and comparatively low cost. If these conditions could be met as well or better in some other way, high specific resistance would be of no importance whatever.

Alloys are certainly inferior to pure metals in some respects. Aside from the molecular rearrangement which may be going on in either the alloy or pure metal, alloys suffer from disintegration and possibly from internal chemical changes which are impossible in pure metals. It is also true that alloys frequently suffer from contact with their surroundings. Manganin, for example, is very easily oxidized, and is even pronounced by some investigators as worthless. Among the pure metals there are several which resist oxidation and other chemical changes admirably. Now if it is possible to obtain a high resistance in the form of a pure metal, and at the same time to retain all the advantages of an alloy, such a resistance ought to soon find favor in the electrical world.

From the results of some work done a few years since by Miss Isabelle Stone,[1] we have reason to believe that metals in the form of

[1] "On the Electrical Resistance of Thin Films," PHYSICAL REVIEW, Vol. 6, pp. 1–16.

thin films upon glass exhibit certain qualities unlike those of the same metals in the ordinary form. Miss Stone reached three conclusions, which may be stated as follows :

1. The electrical resistance of such films as she investigated decreases in value quite rapidly for a short time, then less rapidly for a much longer time.

2. The higher the resistance of the film, the more rapid is the decrease in value.

3. For very thin films, the ratio of the measured resistance to the calculated resistance is high.

Miss Stone's research included silver films only, and these were deposited from aqueous solutions by what is known as the Rochelle salt method ; but her work is suggestive of a very large field of research, if other methods of deposition could be used.

As early as 1877 Professor A. W. Wright[1] of Yale, described a method of depositing thin metallic films upon glass by electrical discharge. Professor Wright produced both opaque and transparent mirrors from a large number of metals, pointed out the difference in rate of deposition of different metals, and suggested that on account of the difficulty of depositing aluminum and magnesium, these metals should be used for electrodes in vacuum tubes in order to avoid the discoloration so common in the neighborhood of the kathode when platinum electrodes are employed.

About two and a half years ago it was my good fortune to learn the practical details of Professor Wright's process in Ryerson Physical Laboratory at the University of Chicago ; and to have the opportunity of preparing a number of mirrors and thin metallic films, with the admirable apparatus designed for this purpose by Professor Stratton and Dr. Mann, of that institution.

DEPOSITION OF FILMS.

The deposition is effected in a vacuum, by a process which may here be included for convenience under the general term kathode

[1] "On the Production of Transparent Metallic Films by the Electrical Discharge in Exhausted Tubes," Am. Journal of Science and Arts, Vol. 13, pp. 49–55.

"On a New Process for the Electrical Deposition of Metals and for Constructing Metal-Covered Glass Specula," Am. Journal of Science and Arts, Vol. 14, pp. 169–178.

discharge,[1] from an electrode consisting of the metal to be deposited.

The necessary apparatus for doing the work advantageously includes a vacuum pump capable of reducing the pressure in the receiver to a few millionths of an atmosphere; an induction coil capable of producing a spark eight or ten centimeters long; an interrupter making a complete break in the circuit, and preferably of high frequency; and a source of electrical energy, capable of furnishing a current of several ampères, at a pressure of not less than fifty volts, if the Wehnelt interrupter is used.

The pump used in the part of the work done in Chicago was a double acting Geissler pump, all glass, and exhausting into a very good secondary vacuum at each end of the stroke, so that the mercury in the main pump was never in contact with the air. This pump is capable of producing a splendid vacuum, but must be rather carefully handled, as the tendency to develop leaks is somewhat aggravated by the fact that the entire apparatus is in continual motion when in use.

In continuing the work at Columbia University, it seemed desirable to construct a pump which should be free from constant danger of developing leaks, and which should be capable of producing the required vacuum rapidly and easily. The Sprengel pump is slow, and the ordinary Geissler pump with its numerous ground joints, valves and stop-cocks, and its rubber connecting tube, is a little uncertain, and at best, somewhat less effective than the necessities of this case require. A Geissler pump was finally decided upon, but not without a determination to eliminate some of its objectionable features.

Figure 1 shows a diagram of the working parts. The right-hand side of the pump, as viewed in the diagram, is in some respects similar to the Bessel-Hagen[2] pump, but simpler. The tube (A) leading from the exhaust chamber to the receiver rises to the height of a full meter above the level of the mercury at (B), when the reservoir (C) is elevated. This feature of the apparatus is used instead

[1] The nature of the process will be considered more in detail hereafter.

[2] "Ueber eine Neue Form der Toepler'schen Quecksilberluftpumpe und einige mit ihr angestellte Versuche," Wied. Ann., Vol. **12**, pp. 425–445.

of a valve to prevent the flow of mercury from the exhaust chamber (D) to the receiver. As the mercury sometimes rises in this tube with considerable momentum, it reaches a point considerably above barometric height, but the bulb (E), two or three centimeters in diameter, arrests any unusually active mercury which might otherwise pass around the bend at the top of the tube. The tube (F) completes the passage to the receiver, by way of the drying chamber (G), which is sealed on with the blow-pipe. There are no ground joints or even mercury seals in any part of the apparatus.

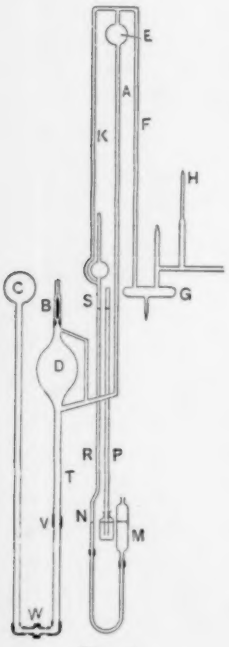

Fig. 1.

The tube (H) is a capillary through which air may be gradually admitted when the receiver is to be opened. The tube (K) leads to the McLeod gauge (L). The reservoir (M) is for the purpose of filling the McLeod gauge. Before beginning to exhaust, the mercury in the gauge tube stands at (N), at the same level as the mercury in the cistern of an independent barometer (P). As the exhaustion proceeds the mercury rises in the tube (R) until it stands at (S), the level of the mercury in the independent barometer. Of course the tube (R) answers the purpose of a gauge until the mercury rises so high that the difference between the two mercury columns is not easily readable. After this the McLeod gauge is used.

The rubber tube which usually connects the reservoir with the exhaust chamber in the Geissler pump is entirely displaced in this case, by an iron pipe with a swinging joint. This improvement originated with Professor Wm. Hallock, of Columbia. The glass tube (T) extends downward from the exhaust chamber as far as (V), where it is securely cemented into an enlargement on the iron pipe. From this point on to the reservoir (C) the mercury passage consists entirely of iron pipe. The swinging joint (W) is a carefully selected pipe union with well polished bearing surfaces moving upon a leather washer. The reservoir is raised and lowered by

simply swinging the pipe, and the rubber tube is thus entirely eliminated.

This pump has now been in use several months, and its behavior is most thoroughly satisfactory. Entire freedom from danger of leaks is a source of inestimable satisfaction in work of this kind. The results obtained are so uniform that, as long as the same receiver is used, it is easy to tell before beginning an operation, just how many strokes will be necessary to produce a certain degree of attenuation in the receiver. So confidently can this uniformity of results be relied upon that the McLeod gauge has become almost unnecessary.

The exact degree of exhaustion which this pump is capable of producing has not been carefully determined, because it has not been necessary to push it to its limit; but the fact that a pressure of one hundred thousandth of an atmosphere may be reached with extreme ease and certainty is evidence that the limit of usefulness of the pump has not been approached.

Notwithstanding the fact that this pump is single-acting and open to the air at one end of the stroke, the necessary vacuum for the deposition of metals can be produced with it in less than half the time required for the same operation with the double-acting pump already referred to.

The induction coil used in the earlier experiments was a large one, capable of producing a 30-cm. spark in air. The coil used later was not more than half as large; but a coil half the size of either of them would be abundantly large for the purpose.

The current interrupter used in the early part of the work was a mechanical interrupter operated by an electric motor in a separate circuit. The speed of the motor was usually about 1,500 revolutions per minute, with two breaks per revolution. This was quite satisfactory except that the deposition of metal would have been more rapid with an interrupter of greater frequency. The Wehnelt interrupter[1] is so well suited to this work that no other has been used since its advent.

The receiver in which the films are deposited is represented in vertical section in figure 2. (*AA*), figure 2, is a rubber stopper through which a glass tube (*B*) enters the receiver. This tube serves at

[1] Elektrotechnische Zeitschrift, Vol. 20, pp. 76–78.

once as an exhaust tube and as a passage way for the kathode wire (*C*). The tube and stopper are securely cemented into the open top of the receiver, with sealing wax (*DDDD*). The lower end of the kathode wire terminates in a thin aluminum tube, just large enough to drive firmly into the lower end of the glass tube (*B*). The kathode plate (*E*) which consists of the metal to be deposited, is supported by an aluminum rod (*F*), which slides into the thin aluminum tube with just enough friction to hold it in place. (*G*) is a heavy aluminum base plate, which serves as the anode, and (*H*) is an additional aluminum plate, which serves as a support for the glass plate (*L*), upon the upper surface of which the film is to be deposited. The dimensions may be taken from the figure as it is drawn to scale.

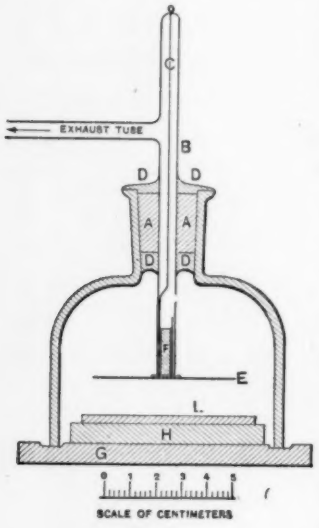

Fig. 2.

When the air is exhausted from the receiver and discharges from the kathode of an induction coil take place from the surface of (*E*), particles of the kathode plate (*E*) are deposited in the form of a brilliant film upon the surface of the glass plate (*L*) and, in fact upon the entire inner surface of the receiver.

The character of the film depends largely upon the rate of deposition, and this in turn depends upon the vacuum, the electromotive force, the current, the frequency of the interrupter and the distance from the kathode to the glass plate upon which the film is to be deposited. If all the conditions are properly adjusted, moderately rapid deposition will produce films of great hardness, density and brilliancy. If the deposition is too rapid, the resulting films will possess these qualities in less degree.

The factors which have been enumerated as affecting the rate of deposition are so intimately related, and so dependent upon each other, that it is quite impossible to discuss them independently. Professor Wright produced beautiful mirrors from a very small

kathode in a 2 mm. vacuum at a 3 mm. distance with a primary electro-motive force of perhaps a dozen volts, but in order to obtain an even distribution of metal over any very considerable surface, he found it necessary to keep the kathode moving over the surface during the process of deposition. It is just as easy to obtain an even distribution of metal over a large surface from a stationary kathode by using a correspondingly large kathode and placing it at a large distance from the glass. This, however, involves working in a higher vacuum and using a higher electro-motive force; and in order that the process may be as rapid, it necessitates using either a stronger current or a higher interruption frequency. A glass surface 5×6 centimeters may be beautifully platinized by placing it at a distance of 12 or 15 millimeters from a kathode plate of similar dimensions, in a vacuum of from .0001 to .00001 of an atmosphere, and operating with a primary current of 5 or 6 ampères at 110 volts, and with an interrupter frequency of about 300 per second.

The two factors, vacuum and distance, are related to each other in a way which demands a more detailed consideration. With a certain fixed vacuum, if the distance from the kathode to the glass plate is much too great, the film will be soft and spongy; while if

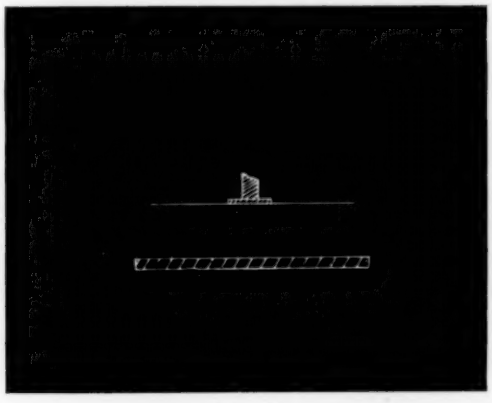

Fig. 3.

the distance is much too small it is almost impossible to get any metal deposited at all. It is found by experiment that the golden mean between these two extreme conditions is attained when the surface of the glass plate is just about in the plane which marks the

boundary between the kathode space and the luminous glow which surrounds it. (See figure 3.) Films deposited at other positions vary very greatly in hardness and density. Some platinum films will scarcely endure the touch of a camel's hair brush, while others can scarcely be removed from the glass by the most vigorous rubbing, or by the action of hot aqua regia.

At the suggestion of Professor Rood an attempt was made to discover the condition of the metal during its transition through the kathode space. A glass plate 5×6 centimeters, was placed at the usual distance, about 15 mm. from the kathode. In the center of this plate was placed a small aluminum stand, supporting a small glass plate about 12 millimeters square, within 3 mm. of the kath-

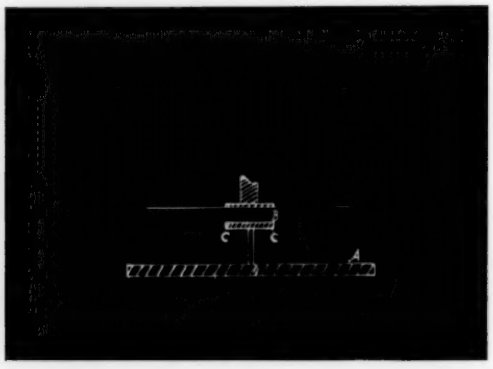

Fig. 4.

ode, as in Fig. 4. The air was then removed from the receiver until the kathode space just reached the surface of the large glass plate (*A*). A platinum film of considerable thickness was then deposited. When the plates were removed from the receiver, it was found, first that the film on the upper surface of the small plate (*B*) was exceedingly thin; next, that the stand had not cast a distinct shadow, but that the film on the large plate under the stand gradually shaded off to a comparatively thin center, as if the particles of platinum had drifted under the stand in considerable quantities. The third and most surprising fact to be noted was that the unprotected corners of the small plate were quite heavily coated on the *under* side.

In the light of these facts there can scarcely be any doubt in regard to the nature of the process. The surface of the kathode is

intensely heated, and particles, probably molecules, possibly smaller particles,[1] are projected into space. These particles radiate from the kathode in the gaseous form until they reach the limit of what is called the kathode space. In other words the kathode space *is* the space in which the metallic matter radiated from the kathode is still in the gaseous state. When the temperature has fallen to a sufficient degree, condensation begins, and we have the visible glow just outside the kathode space—a miniature snow-storm.

The very hot metallic gas near the kathode will not easily adhere to and condense upon the glass, and the comparatively cool "*vapor*," if we may use the term, condenses in rather a loose, soft, spongy layer. It is on this account that the best mirrors are formed just in the *edge* of the kathode space, as described on page 47.

This view is supported by the fact that the visible glow rises to the edge of the small stand as represented at (*C*), in figure 4. The metallic gas flowing over the surface of the stand is cooled somewhat, and the snow-storm therefore begins at a shorter distance from the kathode in this region than in the free space in the other parts of the receiver. Further evidence is offered in the fact that the glow around the stand is more conspicuous at first than it is after the stand itself has become somewhat heated.

The process seems to be simple distillation, in which the vaporization of the kathode depends *largely* upon its electrification.[2] That the process is not *entirely* dependent upon electrification, however, is evident from the fact that selenium, which *boils* at about 700 degrees, is deposited thousands of times more rapidly than platinum.

It is a noteworthy fact that when a rectangular kathode is used in a cylindrical receiver, the deposit on the sides of the receiver is thickest at small areas opposite the corners of the kathode. This is not because the distance is less, but because the surface density of the charge is greater at these points.

[1] J. J. Thompson, in Phil. Mag., Dec., 1899, "On the Masses of the Ions in Gases at Low Pressures," says that in case of the stream of negative electrification which constitutes the kathode rays, there are reasons for thinking that the charge on the ion is not greatly different from the electrolytic one, and that in the former case we have to deal with masses *smaller than the atom*.

[2] See Sir WilliamCrookes "On Electrical Evaporation," Electrical Review, Vol. 28, pp. 796–798 and 827, 828.

In conducting this process, it is not an easy matter to keep the vacuum at a fixed value during the first part of the experiment. When the circuit is first closed, the effect is to drive off the residual air and occluded gases. This produces a change of pressure in the receiver, which is quite rapid at first, but less and less rapid as the process continues. The rate at which the first change takes place depends largely upon the nature of the kathode, the condition of the atmosphere to which it has been exposed and the length of the exposure.

Under what might be called ordinary conditions, when a platinum kathode is used, it is not well to allow the current to continue more than a few seconds when first turned on, without stopping to observe the condition of the vacuum. If the vacuum is allowed to fall below .0001 of an atmosphere[1] there is danger of the film being rather soft. The film in this condition will not adhere well to the glass. Hence the importance of being particularly careful about the vacuum at first.

The curves (*A*) (*B*) and (*C*) in figure 5, represent the rate of deterioration of the vacuum during the rapid part of its change. The vertical portions of the curves represent intervals during which the coil is not in operation, but the pump is being used to improve the vacuum. These curves are all three for platinum films. Curve (*A*) represents an extreme case in which, after depositing a film, the receiver had been quickly opened, the film removed, new glass inserted and the receiver again sealed and exhausted—all within a few minutes. Curve (*C*) represents another extreme case in which both the kathode and the inside of the receiver had been exposed to the air for a long time and under very unfavorable conditions. Curve (*B*) may be said to fairly represent the deterioration of the vacuum during the first ten minutes of the process under average conditions. It must be understood that in all these cases, the minutes represented on the axis of abscissæ are not consecutive minutes, but minutes during which the process of deposition is actually going on.

Usually after ten or fifteen minutes of *actual deposition*, the condition of the vacuum does not change much and the deposition

[1] With the distance, voltage, etc., as stated on page 47.

may then go on continuously if the strength of the current used is not such as to heat the receiver excessively. After this stage has been reached, if the process is discontinued for half an hour or so, if there has been no perceptible leakage, the vacuum is found to

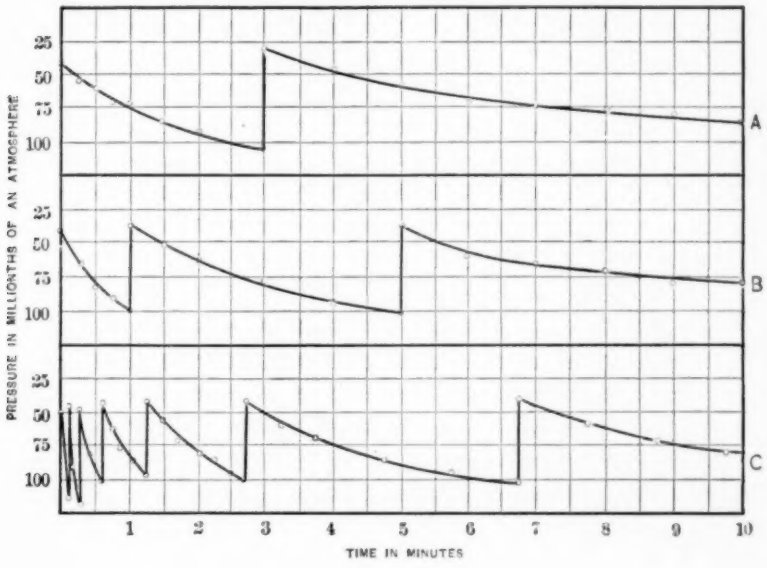

Fig. 5.

have improved considerably on account of the fact that most of the residual gas in the receiver has been occluded by the kathode and film. If the current be again started, however, the vacuum will soon fall to about its normal value.

Electrical Properties of Films.

Besides possessing splendid reflecting surfaces, such as commend them strongly for all high grade optical work, and besides displaying the colors of the different metals by transmitted light, and the selective absorption of different thicknesses of the same film, metallic films deposited in accordance with the method here described possess certain advantages as electrical resistances.

Without an exact method of determining the thickness of a film it is impossible to make an exact estimate of its specific resistance; but even with only an approximate determination of thickness, the

very rapid increase in resistance corresponding to diminishing thickness is so conspicuous as to leave no room for doubt that in *thin* films the ratio of the measured resistance to the calculated resistance is high. For example, a platinum film 5 cm. long, 15 mm. wide and .0002 mm. thick has a resistance of only a few ohms, while a film apparently about one tenth as thick [1] has a resistance of several hundred ohms, and a film probably about one hundredth as thick, has a resistance of hundreds of thousands or even millions of ohms.

There is plenty of room for further investigation in this direction, but even as the matter stands, it seems quite unnecessary to use alloys for the purpose of obtaining high specific resistance.

Quite early in the history of this investigation it was observed that during the heating and cooling of certain resistances, for the purpose of artificial ageing, the resistance changes were not as great as the temperature changes seemed to warrant, and in one noteworthy case the resistance change seemed to be in the wrong direction. Accordingly, the temperature coefficient of this particular film was carefully determined.

This film was deposited April 4, 1898. During the preliminary treatment for bringing it to a condition of stability, its resistance was measured quite frequently. The measurements were made at temperatures differing by a few degrees, and, even during the first few days, while the changes in resistance were quite large, there was at least an indication that the temperature coefficient was probably negative.

On April 21st, the temperature of the film was reduced 19 degrees, and the reduced temperature was kept constant for several hours. This fall of temperature was accompanied by an *increase* of resistance amounting to a little more than 6 ohms, the total resistance of the film being a little less than 2,300, though it had not yet reached its final value. When the temperature of the film was raised to its former value, the resistance *fell* about 4 ohms. The discrepancy between the 6 ohms rise and the 4 ohms fall was due to the fact that the process of artificial ageing was not yet finished, but there was no longer any room to doubt that the temperature coefficient of this film was negative.

[1] Methods of determining approximate thickness will be discussed later.

On April 27th, when the resistance had become more nearly constant, this film was provided with platinum terminal wires, and sealed into a glass tube from which the air was afterwards exhausted to about .001 of an atmosphere, the tube being then hermetically sealed.

Its record on the last three days of the month was as follows :

Date.	Temperature.	Resistance.
Apr. 28. 1898	23.2 degrees.	2284.1 ohms.
" 29 "	2.0 "	2290.5 "
" 30 "	23.1 "	2284.2 "

The temperature coefficient calculated from these results is — 0.00013.

Even at the date of these measurements, this film was not *perfectly* seasoned, but the subsequent changes in resistance were very slight, and after October 1, 1898, no changes whatever could be detected except such as were in exact harmony with the above named temperature coefficient.

In November, 1898, Mr. F. B. Fawcett's very interesting article "On Standard High Resistances" appeared.[1] In this article Mr. Fawcett points out the early rapid decrease in the resistance of a film and the importance of artificial ageing. He also indicates a method of standardizing the resistances and a method of artificial ageing; but in determining the thickness of the films, he assumes that the specific resistance is the same for all films.

The Electrical Contacts.

In these experiments for a considerable length of time the contacts between the films and their terminal wires were an unfailing source of annoyance. Clamps were used at first, but they proved to be untrustworthy. A piece of tin foil or silver foil may be clamped to a *thick* film and the contact may be as good as any clamp contact, which is not saying very much. Of course bright metal plates may be very successfully clamped together for temporary connections, but even the best of clamp connections can hardly be considered first class permanent contacts on standard resistances. Furthermore if a resistance having clamped contacts be boiled in oil

[1] Phil. Mag., 5, Vol. 46, pp. 500–503.

or paraffin for a number of hours, as in the process of artificial ageing, there is a strong probability that the insulating material will get into the joints, especially if the coefficient of expansion of the clamp is greater than that of either of the materials held together.

Aside from all this, when we consider making contacts with a *thin* film, there is an additional difficulty. These films are rather delicate and will not endure the rough usage to which *thick* films, wires and metal plates may be subjected. If the slightest crack be produced in the film, by the pressure of the clamp, the crack will expand and contract under the influence of temperature changes, and in this way a variable resistance will be introduced into the circuit. It was for these reasons that a number of attempts were made, several of which resulted in perfectly satisfactory methods of making electrical contacts with even the thinnest of the films.

The first attempt, which was not altogether successful, is represented in figure 6. (AA) are strips of tin foil or platinum foil fastened to the glass with shellac varnish and thoroughly baked. The film (B) is deposited afterwards over the entire surface of glass and foil, and if thick enough, the continuity between the film on the glass and the film on the foil is perfect. If, however, the film is thin, there is lack of continuity at the edge of the foil. The method may be used even for thin films by covering the edge of the foil with gold leaf before the film is deposited. This makes a joint which is electrically good, but poor mechanically.

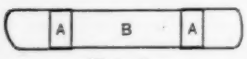

Fig. 6.

The second method is satisfactory in every respect and is applicable to films of any thickness. This method is represented in figure 7. In this case the film is deposited first, and over the entire surface of the glass. The receiver is then opened and the portion (B) of the film is covered. The receiver is again closed and exhausted and the deposition of metal is simply continued until the portions (AA) are very thick. We then have a continuous film, as thin as we please, for a high resistance, but with *ends* as thick as we please for making connections. The fine copper wires (CC) are then wound on and permanently secured by electrolytic deposition of copper on them.

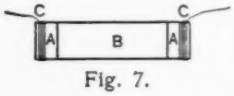

Fig. 7.

The resulting contact leaves absolutely nothing to be desired, but the process is rather tedious. A *thin* film may be produced in a few minutes, after the receiver is exhausted, but these *thick ends* require hours. It was on this account that a third method was devised. Figure 7 represents this method as well as the preceding one. In this case, the ends of the glass plates are first immersed in a silvering solution, and thick films of silver (AA) are deposited upon them by any of the well-known methods. As the plate stands in the silvering bath the portion which is below the plane of the surface of the bath becomes heavily coated; but there is a small area just above this plane, which receives silver from only the small amount of liquid which rises above the plane of the surface of the bath by capillary action. For this reason, the film, however thick it may be, always terminates in a very thin edge so that the film (B), which is afterwards deposited is perfectly continuous over the entire surface of the glass and silver. The copper wires (CC) are secured in this case in the same way as in the preceding case. If the silvering of the ends had to be done one piece at a time, this method would have very little advantage over the preceding one, but, as a large number of plates may be placed in the silvering bath at the same time, the amount of time spent in silvering the ends of the plates is very small.

Fig. 8.

The finished film, as it appears in figure 8, may be introduced into a Wheatstone bridge circuit, or into any other electric circuit for which it is suitable, just as if it were a coil of wire. It has, however, the advantages of being non-inductive and practically without capacity.

(To be continued.)

A METHOD OF STUDYING ELECTROSTATIC LINES OF FORCE.

BY F. J. ROGERS.

SINCE the days of Faraday and Maxwell much has been made of electrostatic lines of force. In recent years this method of dealing with electrostatics has been extensively introduced into elementary text-books. Now, without experimental illustration these lines of force represent an extremely abstract conception, arrived at only by processes of reasoning and imagination.

To make the existence and reality of these lines of force evident to the average student, an experimental demonstration is necessary. This is also highly desirable even for those students who are gifted with acute reasoning powers and a well-trained imagination.

I have tried a number of methods for obtaining this result with but partial success. The method described by several writers of suspending an elongated dielectric by a fiber in the field to be studied is very unsatisfactory. The forces are weak because the dielectric constant is always a small number; the case being very different from the analogous one of a piece of soft iron in a magnetic field, where the permeability is large. Furthermore, the dielectric "needle" is likely to be attracted bodily to a charged conductor and stick to it.

I finally tried a very simple method which is quite satisfactory; so much so, that by its aid students may trace out electrostatic lines with almost as great a facility as they can magnetic lines.

The method consists in the use of an "electrostatic needle" mounted on a rigid axis. In order to increase the forces brought into action, each end of the needle is provided with a small gilt pith ball.

I have used two forms of this apparatus. The first consists of an aluminum wire pierced at the center and provided with pith balls at its ends. This "needle" is mounted on a horizontal axis con-

sisting of a pin stuck into an ebonite handle (see Fig. 1). When this needle is brought into a fairly strong electrical field, the two balls acquire, by induction, opposite electrical charges, and the forces acting on these cause the needle to point along the lines of force.

With the aid of this electrostatic needle it very easy to show: (1) That lines of force are normal to the surface of a charged conductor (an elongated cylinder for example); (2) that lines of force are *not* normal to the surface of a hard rubber rod one end of which has been charged by friction; (3) that in the case of two oppositely charged conductors placed near each other the lines of force issue from one conductor, curve over, and end on the other; (4) that in the case of two similarly charged conductors, the lines of force from each leave it normally and pass out into space, none of them passing from one conductor to the other.

Fig. 1.

A much more useful form of electrostatic needle is constructed as follows: In place of the aluminum wire a small rod of hard rubber is used. It is mounted on a shaft and provided with pith balls as in the first form.

The pith balls are oppositely charged by contact with the poles of an electrical machine. While so charged the electrostatic needle behaves in an electrostatic field just as a magnetic needle does in a magnetic field.

In addition to the experiments already described, this form of electrostatic needle shows the positive direction of lines of force; consequently, in passing a neutral point in the field, the direction in which the needle points is reversed. It is very useful for testing the field about the different elements of an induction machine. For example: Suppose the neutralizing-rod of a Voss machine is removed while the machine is running. Under favorable circumstances the inductors become charged to a high potential, then lose their charges, and acquire an opposite one. This cycle may be continuously repeated every few seconds. To the experienced operator this periodic change is rendered obvious by the change in the appearance and sound of the brush discharge. That the polarity of the machine is

being periodically reversed may be made evident to anyone by holding near one of the inductors a polarized electrostatic needle. The needle reverses its direction every time the polarity of the machine is changed.

The pith balls of the electrostatic needle do not remain charged for any great length of time. These charges are lost quite rapidly in the neighborhood of a brush discharge.

NOTES ON THE PREPARATION OF RETICLES.

By David P. Todd.

TO the physicist more often perhaps than to the astronomer occurs the problem of preparing a reticle of unusual design. If only straight lines are desired, nothing surpasses the fine web of the spider, which is easy to thread; or if regular intervals are requisite, they may be graduated on glass with a ruling engine.

When, however, a complicated reticle becomes necessary for any purpose, or one with curved lines, letters and figures, the spider cannot help us, and execution of the design on glass becomes very difficult, and too expensive. But a photographic method is still very serviceable.

Three of the instruments devised for the Japan Eclipse Expedition of 1896, described elsewhere,[1] required circular reticles about 0.2 in. and 0.5 in. in diameter, with careful graduation around their circumference.

First, an exact original was draughted on Bristol board with India ink, making the circle about 9 inches in diameter, and including all the lettering, with the finest gradation necessary. A sharp negative of this was then made, reduced to about one inch; and from this the required number of positives were prepared, with a farther reduction suitable in each case to the focal length of the telescope in which it was used. The proper reduction may be found by measuring the focal length, and calculating the linear value corresponding to the arcs intended.

By using the thinnest cover glass, and employing the wet collodion process, a fine grained reticle of high transparency is easy to obtain; one, too, which will bear a high degree of magnification.

Observatory House, Amherst College.

[1] *Monthly Notices Roy. Astron. Society*, LVIII. (1898), 296; *Proc. Am. Assoc. Adv. Science*, XLVI. (1897), 178.

NEW BOOKS.

Electromagnetic Theory. Vol. II. By OLIVER HEAVISIDE, F.R.S. London, *The Electrician* Printing and Publishing Company, limited, 1899.

The second volume of Mr. Oliver Heaviside's "Electromagnetic Theory" forms a suitable continuation to the first volume of that work as well as to his "Electrical Papers," and this will be welcomed by all students of his writings. The present volume is mainly occupied with detailed mathematical calculations on the propagation of electrical disturbances along circuits of various kinds, a subject which Mr. Heaviside has made particularly his own. Those who wish to gain a general knowledge of the consequences of Maxwell's theory will find them explained with great clearness and with very few mathematical symbols in the first volume of this treatise; these descriptions are not the work of a mere mathematician driving blindly some analytical machine, but of a man possessing a singular insight into the physical processes involved in the propagation of electromagnetic waves.

A university education, and the help which it brings by intercourse with men of like pursuits, are great aids as many mathematicians of the present day would acknowledge; but these things have not come to Mr. Heaviside. Attracted, perhaps, to electrical studies by the practical work of telegraphy, he has gone steadily on without these aids, always keeping the problems of electromagnetism before his mind, and striving with marked success to improve the mathematical methods of attacking these problems. By patient investigation, extending over many years, he has gained a remarkable knowledge of the detailed results of Maxwell's theory and a special skill in the mathematical analysis appropriate to the subject, winning a very high position among the ranks of investigators and awaking a feeling of curiosity among physicists to know something of his work. The average professional physicist spends so much time in teaching that he is not often able to study books requiring very close application, such as those of Mr. Heaviside. This is sad, but it is true. Such men often confess that one of the things they are always intending to do is to make a thorough study of Mr. Heaviside's work; they are sure there is a great deal in it.

The first chapter of this volume is for the most part occupied by a mathematical account of those results in the theory of the diffusion of

heat which shed some light upon the age of the earth. The following chapter deals with other diffusion problems, these now referring to the spreading of an electric charge along a cable on the [impossible] assumption that the effect of the inductance (self-induction) of the cable is neglible. Solutions of many problems are obtained, some of the most interesting cases being those in which various arrangements of resistances, capacities and inductances are placed between the cable and the source of electrical energy.

The third chapter is perhaps the most important part of the book, dealing, as it does, with the propagation of electrical waves along a real circuit. The twin wire circuits of long-distance telephone lines have four characteristics of which account must be taken, viz: R the resistance, L the inductance, S the permittance (capacity) and K the leakage conductance from wire to wire, all per unit of length of the circuit. The fundamental problem and its solution may be described as follows: Suppose that at any place on such a circuit, the two poles of a battery are put into connection with the two wires for a very short time τ. Then in either direction there will go out two disturbances, each of thickness $v\tau$, travelling with the velocity $v = (LS)^{-\frac{1}{2}}$, the velocity of light in the medium between the wires. These disturbances shrink as they go, the attenuating factor being $\varepsilon^{-\rho t}$ where $\rho = R/2L + K/2S$. But this is not all. For along the whole of that portion of the circuit which lies between these two waves there is a disturbance which, beginning by being comparatively small, ultimately (in practical cases) drains all the energy out of the two original waves. These separate parts of the disturbance Mr. Heaviside calls the "head" and the "tail." The ratio of the total disturbance in the "tail" to that in the "head" at any time t after the starting of the waves is $\varepsilon^{\sigma t} - 1$, where $\sigma = R/2L - K/2S$; hence σ may be taken as a measure of the distortion of the signals. Thus it is plain that for effective signalling both ρ and σ must be small. With R and K both zero, ρ and σ vanish and the circuit is a perfect one transmitting signals without either attenuation or distortion. But all circuits have wires of finite resistance, and thus this ideal can never be realized in practice. But even though the attenuation cannot be got rid of, it is possible to cause the distortion to vanish by making $R/L = K/S$; in this case there is no "tail" to the wave, and the signals, though shrunken, are just as sharp on their arrival at their destination as when first despatched. But, with the values of R and L obtainable in practice, the shrinking would be so excessive that next to nothing would be left of the waves after they had gone a little way. Though (on this account) useless in practice, this "distortionless" condition of the circuit is very useful in theory, for it forms a sort of half-way house between the perfect circuit, in which both R and K are zero, and the circuit in which all four

characteristics have any given values. Now in all practical circuits K/S is less than R/L, and depends simply upon the insulating and dielectric qualities of the medium between the wires, being independent of the size of the wires and of their distance apart. Thus, when this medium has been chosen the only way left for making ρ and σ small, and thus obtaining good signalling, is to make the resistance R small and the inductance L large. It is because it is the magnetic part of the energy which is wasted by the resistance of the wires that it is a help to the signals to increase the magnetic energy by increasing the distance between the wires.

These elementary facts can be deduced by very simple mathematics, being merely examples of the law of compound interest, and thus may quite well be taught to junior students. The detailed examination of the exact form of the "tail" of the wave is a matter of greater difficulty. The result was stated by Mr. Heaviside in 1887 ("Electrical Papers" Vol. II., pp. 150) and proof was given a little later; in the present volume several methods of proof are give. The formula involves Bessel's functions.

This chapter contains a great many of the mathematical weapons necessary for attacking the problem of finding the form of the wave due to any given impressed voltage applied at the end of the circuit; it will be read with great interest by those who are interested in Bexsel's functions. The formulæ are generally complicated; this is perhaps largely due to the fact that the series which make their appearance have never been studied and thus are without any simple abbreviated forms. Even $e^{mt}\sin nt$ will appear complicated when expressed in a series of powers of x. This chapter and those parts of the "Electrical Papers" which deal with the propagation of waves along wires are deserving of the closest study.

The last chapter, devoted to a brief discussion on generalized differentiation and divergent series, will, it is hoped, attract the attention of professed mathematicians. An unprofessional mathematician must be content to remark that even if the mathematical methods are largely experimental, they may still be none the worse for that. It has happened before now that things have been discovered by men who have gone out of the beaten track and have made experiments. These subjects were considered by the author in three papers on "Operators in Physical Mathematics" communicated to the Royal Society. Two of these were read and published; the third was not read. Those who are interested in the theory of the motion of electrically charged bodies will regret that this third paper (from whatever cause) made no further progress. For by the methods therein developed Mr. Heaviside has obtained the solution for the motion of a charge at a speed greater than that of light; it is to be hoped that this incident will not permanently deter Mr. Heaviside from publishing the solution.

In one of the appendices Mr. Heaviside makes some interesting remarks on the motion of a charged body at a speed equal to or greater than that of light. The fact that the electromagnetic energy of a charged body in *steady* motion at the speed of light is infinite has led some who have considered the problem to conclude that it is impossible to make a charged body move at a speed greater than that of light. Mr. Heaviside shows that this conclusion is unwarranted. The error has arisen because the manner in which the steady state is reached has not been duly considered. When a charged body has been set in motion at speed u a force must be applied to it to keep its speed constant, the work done by the force appearing as electromagnetic energy in the field. This force continually decreases as the time increases and after an infinite time vanishes, unless u is greater than the speed of light. Mr. Heaviside states that the force does not tend to vanish when the speed is that of light, but a simple calculation shows that in this case when the body—a sphere of radius a carrying a charge Q—has travelled over a distance vt great compared with its dimensions the force required to keep its velocity constant approximates to $\frac{Q^2}{2avt}$. The force is thus (except near the beginning of the motion where this approximation may not apply) inversely proportional to the time which has elapsed since the motion began and hence vanishes after an infinite time.

A whole science of electrical dynamics is waiting for development; some of its elementary problems may be briefly indicated.

1. The force required to maintain a charged body in uniform motion in a straight line.

2. The acceleration of a charged body due to a uniform force.

3. The impulse required to give a charged body any given velocity.

4. The radial and tangential forces required to maintain the uniform motion of a charged body in a fixed circle. [The unpublished researches of the Rev. P. E. Bateman, of Tonbridge School, England, may possibly give some information on this problem.]

These problems, rendered specially interesting at the present time by the experimental work of J. J. Thomson and others on the behavior of the ions in vacuum tubes, are well worthy of the attention of those who have the mathematical skill necessary for their solution.

It is impossible in a review to give any adequate account of the author's work. A close study of the book itself is necessary. If difficulties are encountered which the reader cannot overcome, let him remember that many students have found Mr. Heaviside very ready to help them in their troubles when they have brought them to his notice.

In conclusion it must be said that *The Electrician* deserves very great praise for providing Mr. Heaviside with the opportunity of publishing

this work. It is not every journal which will devote large portions of its space to matter which it is sometimes (falsely) imagined that no one reads.

G. F. C. SEARLE.

CAVENDISH LABORATORY, CAMBRIDGE, ENGLAND.

Electrical Testing for Telegraph Engineers. By J. ELTON YOUNG. New York, The D. Van Nostrand Company; London, The Electrician Company (no date). Pp. 249.

In this work the author has well carried out his purpose of supplying the want felt by telegraph engineers (more particularly submarine cable electricians) of some book devoted exclusively to their work, comprising both a more complete exposition of previous knowledge and a fuller account of its latest developments than previously published. Measurements of conductor resistance, insulation resistance and capacity, tests for disconnections, earths and contacts and similar topics receive systematic and satisfactory treatment, with particular view to the practical utility of the methods described. The work will likewise be found instructive for other readers than telegraph engineers, although not so well suited for those not familiar with the technical details of cable testing. For such readers the subject is not so clearly set forth as it might well be, and the use of technical abbreviations detracts from its utility as a work of reference. For example we note such headings as "I. C. Tests during Manufacture," and read that "observations of the I. C. are made along with those of the D. R. and C. R." The pages bristle with these and other abbreviations, as G. P., N. R. F., and E. C., and it commonly occurs that an explanation of the abbreviation is not to be found, at least not in the preceding pages.

The book is of unquestionable value to engineers of submarine cable, telegraph and telephone lines, and will prove of some use to those who have the testing of light and power cables.

F. BEDELL.

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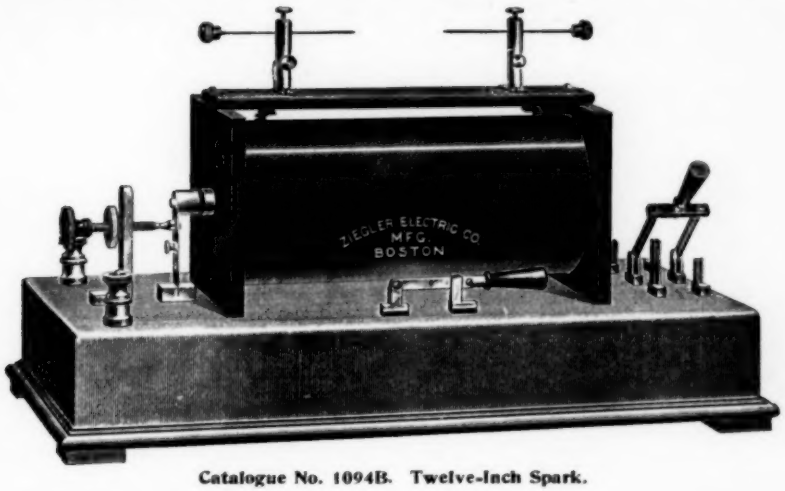

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